USE OF TIME–FREQUENCY REPRESENTATION FOR TIME DELAY ESTIMATION OF NON STATIONARY MULTICOMPONENT SIGNALS

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ABSTRACT

In this paper, we propose three methods for the TOA and TDOA estimation, based on the Choi William Distribution (CWD), and suitable for single as well as multicomponent non stationary signals. The CWD exhibits some very interesting properties, which are exploited for the TOA estimation and which are discussed in the paper; moreover, it turns out to be almost insensitive to even large amounts of noise. The proposed methods are validated by means of numerical examples, which point out their effectiveness in terms of mean value and standard deviation of the estimated TDOA's

1 Introduction

The estimation of Time of Arrival (*TOA*) and Time Difference of Arrival (*TDOA*) between the signals received at two (or more) sensors represents an important problem in several fields, such as sonar, radar, bio-medicine, geophysics. Various methods have been proposed in the case of stationary both single and multicomponent signals [1]. In this paper, we propose a class of methods for the time delay estimation in the case of *non stationary multicomponent signals*, based on Time Frequency Representations (TFR's) [2]. Among all the TFR's, we have selected the Choi-Williams Distribution (CWD) [3], due to the fact that it enjoys properties which reveals particularly useful for the TDOA estimation.

In Sect. 2 a general description of bilinear TFR's is given, with particular attention on the CWD. In Sect. 3, we propose three methods for the estimation of TDOA's, and discuss their applicability to both single and multicomponent signals. In Sect. 4 the proposed methods are validated by means of numerical examples.

2 Bilinear TFR's and the CWD

The TFR's have been developed as an attempt to adequately describe the distribution of the signal energy simultaneously in time and in frequency. Clearly, the capability to localize a signal in the (t, f) plane can be exploited to estimate TOA's and TDOA's of different signals.

In many respects, the Wigner-Ville Distribution (WVD) can be considered the prototype of all the TFR's, as all the other ones have been developed to overcome some drawbacks of WVD and to generalize the method.

It is known [2] that the WVD of a signal $x(t)$ satisfies a large number of properties, among which it is worth mentioning: it is invariant with respect to time shifts, and it preserves the signal energy These properties are of major importance for the application at hand, as the signal representation in the (t, f) plane, obtained with the WVD, permits to “correctly” localize the signal. On the other hand, the major drawback of bilinear transforms (and, in particular, of the WVD) is the presence, in the (t, f) plane, of interference or crossterms (IT in the following) localized in time intervals where no signal energy is actually present. The cross terms can impair the performance of algorithms for the TDOA and TOA estimation. Therefore the WVD cannot be used, and other TFR's must be selected, with good performances in terms of time selectivity, and with reduced cross terms.

Time invariant TFR's form the so called Cohen class [2]. A general formulation of these transformations is

$$T_x(t, f) = \int_{t'} \int_{f'} \psi_T(t - t', f - f') W_x(t', f') dt' df'$$

where the kernel function $\Psi_t(t, f)$ determines the distribution and its properties. For the time delay estimation a proper choice of the kernel (in the Fourier transformed domain) is

$$\Psi_T(\tau, \nu) = e^{-\frac{(2\pi\tau\nu)^2}{\sigma}}$$

The obtained TFR is known as *Choi-Williams Distribution* (CWD) [2]. A discrete time version of this transformation, described in [2], has been used to obtain the results described in this paper.

The most useful properties of CWD for the TDOA estimation can be summarized as follows.

Property P1: The CWD is invariant with respect to time shifts. This is obviously necessary for the TOA and TDOA estimation.

Property P2: The CWD is invariant with respect to frequency shifts. This allows one to select the dominant frequency of a signal at a given time instant.

Property P3: The CWD allows the correct evaluation of the istantaneous frequency at each time istant.

Property P4: The CWD allows the correct evaluation of the group delay at each frequency. This property represents a useful theoretical basis for the TOA (TDOA) estimation.

2.1 Group delay property

According to Cohen, [2], the first conditional moment of time (at a given frequency) is defined as

$$t_x(f) = \frac{\int_t tT_x(t,f)dt}{\int_t T_x(t,f)dt}$$

This quantity represents the barycentric time instant of a generic TFR $T_x(t,f)$ at a given frequency. It can be shown that, for the CWD, $t_x(f)$ is exactly the group delay at that frequency. This property can be extended also to multicomponent signals, and can be effectively exploited. In fact, we can associate the TOA at a given frequency to the time barycenter of the CWD, i.e. the group delay, at that frequency. As a consequence, a TDOA estimation at that frequency can be obtained as the difference between the TOA's evaluated respectively on the TFR corresponding to the signal at the two sensors.

This concept can be better explained by means of an example. In Figs. 1 and 2, the time tracks of the signal at two different sensors are shown, along with the corresponding CWD's at the two sensors. As expected, the property of barycentric frequency is satisfied for every section at constant time. The time invariance is also satisfied and the time barycenter of the CWD corresponds to the applied delay. Moreover, an oscillating signal in the time domain corresponds to a (almost everywhere) positive valued pulse in the transformed domain, and this can be exploited for classical correlation analysis.

In the previous example the two signals have the same frequency components and different time location. It is also possible, working at different frequencies, to obtain separation of signals which are very close to each other in the time domain.

2.2 Robustness of the CWD in the presence of noise

The CWD reveals almost insensitive to even large amounts of noise. In Fig.3 the signal of Fig.1 corrupted with a high level of white noise is represented, along with its CWD. The denoising effect is evident.

The robustness of the CWD with respect to noise can be explained intuitively. First of all, the CWD can be seen as a version of the WD, filtered by means of a 2–D Gaussian window; this window is able to select narrow

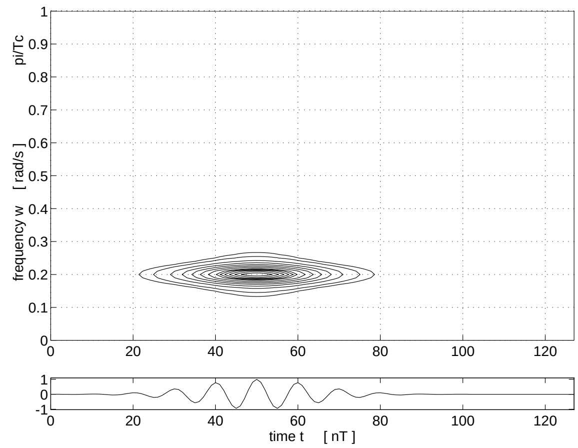

Figure 1: Signal at the first sensor and its CWD

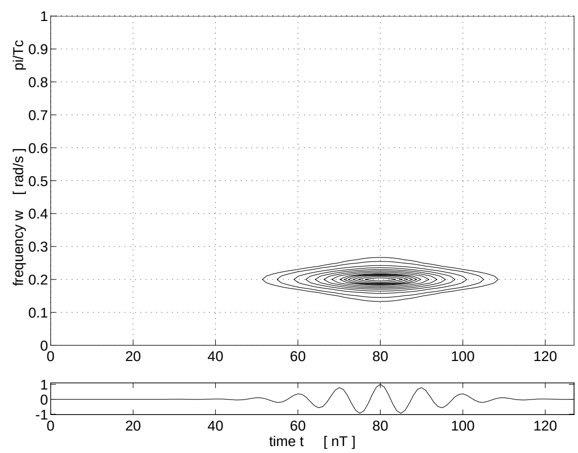

Figure 2: Signal at the second sensor and its CWD

regions of the time–frequency plane, so achieving an effective noise rejection outside this region. Moreover, a noise rejection capability is intrinsic in all bilinear distributions, due to their correlative structure.

3 TOA and TDOA estimates based on CWD

We present three methods which have been tested for the estimation of TDOA's in the case of a two component signal.

3.1 Correlation method

The correlation method is based on the fact that, in the time-frequency plane, it is possible to gain information about the signal behaviour in the time domain, by examining its TFR sections at constant frequency. As a consequence, the estimation of TDOA's is obtained correlating fixed–frequency sections of the TFR corresponding to the signals at the two sensors. This meth-

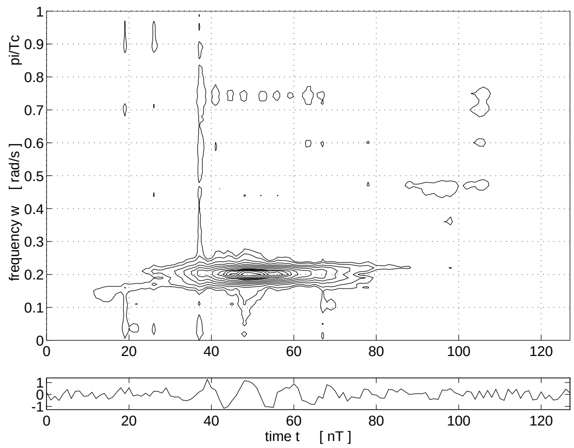

Figure 3: Denoising effect of the CWD

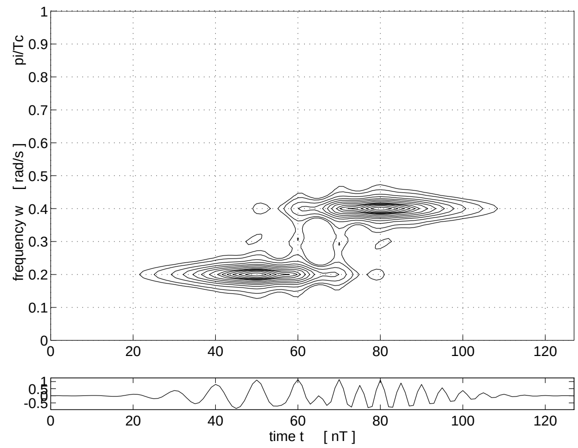

Figure 4: Two component signal and its CWD

ods represents an elegant solution in the case of signals with several components which are not separable in the time domain. Moreover, it makes it possible to estimate the TDOA's for single frequency values.

3.2 Barycenter method

This method is based on the same concept of the previous one, but it makes use of Property P4. In fact, the barycentric time of the TFR at a fixed frequency is equal to the group delay at that frequency, and this can be exploited associating the TOA of a given signal, at fixed frequency, to the barycentric time of its TFR at that frequency, i.e. to the group delay. Therefore, TDOA estimation at a fixed frequency can be obtained for a signal reaching two sensors, as the difference between the barycentric times evaluated on the TFR's of the signals at the two sensors. Compared with the correlation method, the barycentric method exhibits slightly degraded performance, as it will be shown in Sect. 4, but it allows one to evaluate absolute time values (TOA's) as well.

3.3 Maxima method

This method is the simplest one, but, besides being able to evaluate absolute time values as well, it offers noticeable advantages in terms on computational efficiency. It makes use of the maxima of the TFR at fixed frequency, as an approximation of the barycentric times. The TDOA's are evaluated as difference of TOA's as in the previous case.

3.4 The case of multicomponent signals

Let us consider now a multicomponent signal obtained as the sum of the two signals shown in Fig.1. In Fig.4 the signal and its CWD are shown.

If we evaluate sections of the TFR's at two fixed frequencies, it is clear from the figure that we are able to separate the two components of the signal. Notice that, also in the case of signals whose components are not separable in the time domain, the TFR's of the two components turn out to be separable in the time–frequency plane. Moreover, the separation of the components can be obtained both in time and in frequency.

If we have two multicomponent signals a two different sensors, the frequency selectivity of the CWD can be exploited to estimate the TDOA of each component of the two signals. For example, if one has to evaluate the time interval between the arrival at the first and at the second sensor of the first component, he can refer to the TFR sections for the two sensors, at $f = 0.2$. Then, the TDOA can be evaluated in three different ways:

1. Correlation method: the correlation between the two sections at hand is evaluated; this is correct, as only one component is involved at the fixed frequency value. Then, the time at which the correlation is maximum is selected.

2. Barycenter method: the barycentric time of the two sections is selected, so yielding the two TOA's; then the TDOA is evaluated as the difference of the two TOA's

3. Maximum method: the TOA's are evaluated as the maxima of the two TFRs

4 Results

In the previous sections, we have proposed three techniques based on the CWD for TDOA estimation, and we have pointed out that such methods are able to work also with multicomponent non stationary signals; moreover, we have underlined the fact that bilinear TFR's, and in particular the CWD, are able to achieve a consistent immunity with respect to noise.

In this section, we present a numeric validation of the proposed methods, and compare their performance, as long as possible, with the classical method of correlation in the time domain. We have performed Monte Carlo simulations for two experiments, and have evaluated the CWD at those constant frequency sections where it exhibits a maximum. We assume the presence of white noise.

The first experiment consists of two single componnt tracks with $TDOA = 2$. In this case it is possible to compare the proposed methods with standard correlation in the time domain. The second experiment involves two 2–component signals with different TDOA's for each component, namely $TDOA1 = 10$, $TDOA2 = 13$. In this case, a comparison with time correlation is no more possible, as this latter does not yield useful results in the case of multicomponent signals.

The results of the two experiments are reported in Tables 1 and 2 in terms of the average and standard deviation of the evaluated TDOA, at various levels of signal to noise ratio (SNR). The following considerations can be drawn:

- In the first experiment, classical time correlation yields the best performance for high SNR values ($> 14dB$). This is vey reasonable, as the correlation works on the time signal directly, without submitting it to any pre-elaboration, which has a cost in terms of the estimation accuracy
- At lower SNR values, the disadvantage of performing a pre-elaboration is overcome by the higher robustness of the CWD–based techniques towards noise; in particular, when the SNR drops below 5 dB, the correlation yelds completely wrong results
- Among the CWD–based methods, the correlation method behaves the best, yielding reasonable estimation also in the worst considered case of $SNR = 0dB$. The barycenter method exhibits a slights impairment with respect to the former, whereas the maximum methods turns out to be more affected by the presence of noise
- In the second experiment, we can see as the CWD offers good estimation capability also in very severe SNR conditions

5 Conclusions

In this paper, we have proposed three methods, based on the CWD, for the TDOA estimation of multicomponent non stationary signals. The proposed methods have been shown to yield good estimates also in the presence of severe noise levels.

S/N (dB)	CROSS-CORRELATION		METHOD A	
	MEAN	STD	MEAN	STD
14	1.996	0.196	2.000	0.461
8	2.300	1.266	2.023	0.983
5	1.670	2.583	1.906	1.199
3	3.230	5.187	1.773	2.101
0	0.253	5.037	1.746	2.520
S/N	METHOD B		METHOD C	
	MEAN	STD	MEAN	STD
14	2.037	0.652	1.933	1.623
8	1.917	1.169	1.928	2.625
5	1.962	1.578	2.033	2.747
3	1.730	2.625	1.035	3.200
0	1.659	2.807	1.566	5.963

Table 1: Mean value and standard deviation (STD) of $TDOA$ (theoretical value 2) of a signal received at two sensors, with different signal to noise (S/N) levels

S/N (dB)	METHOD A		METHOD B	
	MEAN	STD	MEAN	STD
14	9.95	0.50	9.93	0.46
8	9.98	1.06	9.92	1.07
5	9.93	1.35	9.91	1.73
3	9.45	2.45	9.61	2.65
0	10.06	2.56	10.03	2.86
S/N (dB)	METHOD C			
	MEAN	STD		
14	9.80	0.80		
8	9.71	1.78		
5	9.42	2.99		
3	9.83	3.65		
0	9.80	4.10		

Table 2: Mean value and standard deviation (STD) of $TDOA_1$ (theoretical value 10) of the first out of two components of the signal received at two sensors, with different signal to noise (S/N) levels

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