NON CAUSAL ADAPTIVE QUADRATIC FILTERS FOR IMAGE FILTERING AND CONTRAST ENHANCEMENT

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ABSTRACT

In image contrast enhancement [2, 4, 5], quadratic and more generally polynomial filters are a very popular class of nonlinear filters. These filters exhibit good performances in terms of visual quality, but present some drawbacks such as the elimination of usefull information when using a fixed filter. In this paper we propose a new family of adaptive quadratic filters, where a weighted filter mask is adaptively determined according to the minimization of a prediction error. This filter is then used to enhance locally the image contrast. The results we proposed point out the improvement provided by these new filters in comparison with recent approaches [4, 2].

1 INTRODUCTION

The main problem in the field of image enhancement is to find operators which are capable to sharpen the details of an image but are reasonably insensitive to noise. Moreover the details enhancement must be stronger in bright regions where human visual system is less sensitive to luminance changes (Weber's Law [3]). Ramponi [4] and Mitra [2] proposed an improvement of the classical Unsharp Masking (UM) method [3] by using a complete quadratic filter to enhance text document, or by introducing a generalization of the Teager's algorithm. These techniques which give high quality enhancement for natural images still suffer from their weak noise robustness and are poorly adapted to periodic textured images, where a fixed filter often eliminates useful information. In order to be less sensitive to noise, Ramponi proposed a cubic filter which allows to perform a sharpening action only if the processing mask is located across the edge of an object [5].

The approach presented in this paper is based on a non causal adaptive quadratic filter, i.e. a filter formed by combining linear and quadratic operators whose coefficients are adaptively determined. The linear part of the filter smoothes the data, while the quadratic part achieves an enhancement in accordance with the characteristics of human vision. The noncausality and the adaptivity of the filter mask allows to take into account all the surrounding pixels highly correlated to the current pixel to be filtered, and then to give better results in the case of periodic textured images.

2 ADAPTIVE NON CAUSAL FILTER MASK

2.1 Salembier adaptive approach

The idea of adapting the filter mask is due to Salembier for adaptive rank order based filters [1]. This procedure consists in defining a search area, and in assigning a coefficient to each possible location. Finally the current filter mask support is obtained by thresholding the set of coefficients : if a coefficient is greater than the threshold, the corresponding location is considered as belonging to the unweighted filter mask. An adaptation process is used to optimize the filter mask support by modifying the set of continuous values of the search area.

2.2 Principle of the new adaptive approach

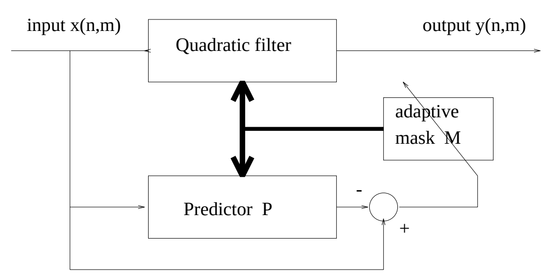

figure 1 : filter's structure

The main difference between our approach and the Salembier approach is that we use a weighted filter mask. We define the non causal filter mask as a weighted window of size $K*L$ centered on the current pixel (n,m). A coefficient $m_{i,j}$ of M is viewed as a level of confidence for the corresponding location to belong to the mask.

$$M = \{m_{i,j} \in [0,1]/i,j \in K*L\} \quad (1)$$

All the pixels (i,j) belonging to to the mask are taken into account according to their weight $m_{i,j}$, and no thresholding is necessary to obtain a binary mask as in the Salembier approach. Our algorithm operates in two

steps : the calculation of the coefficients $m_{i,j}$ and the quadratic filtering implementation. The general structure is given in figure 1 : the coefficients are adaptively calculated using a prediction error, and the quadratic filter is performed using these coefficients.
According to the choice of the predictor and the quadratic filter, one can derive various schemes of implementation from the generic filter structure presented in figure 1.

2.3 Adaptation process

As described in figure 1, the set M of coefficients $\{m_{i,j}\}$ is calculated in order to minimize a prediction error. The predicted value p(n,m) is computed as follow :

$$p(n,m) = \frac{\sum_{i,j \in M^*} m_{i,j}\, x_{i,j}}{\sum_{i,j \in M^*} m_{i,j}} \tag{2}$$

with $x_{i,j}$ pixel at position (i,j) in the mask, and $M^* = M \backslash (n,m)$.
p(n,m) is in fact a weighted mean over the mask M^* (excluding the current position (n,m)). Two prediction error criteria J1 and J2 are then considered, the Mean Square Error (MSE) and the Mean Absolute Error (MAE).

$$J1 = E[(p(n,m) - x(n,m))^2] \quad (MSE) \tag{3}$$
$$J2 = E[|(p(n,m) - x(n,m)|] \quad (MAE) \tag{4}$$

A steepest descent algorithm [6] is used to update the set of coefficients at step k in order to minimize the corresponding prediction error criterion (J1 or J2) :

$$\mu_{i,j}^{k+1} = m_{i,j}^{k} - \lambda \frac{\partial J}{\partial m_{i,j}} \tag{5}$$

where $\mu_{i,j}^{k+1}$ are temporary values which will be used later to compute $m_{i,j}^{k+1}$ by using a scaling transformation.
So, according to the choosen criterion, we obtain two updating algorithms :
MSE :

$$\mu_{i,j}^{k+1} = m_{i,j}^{k} + 2\lambda_{MSE} (p(n,m) - x(n,m))\left(\frac{p(n,m) - x_{i,j}}{\sum_{i,j \in M^*} m_{i,j}^{k}}\right) \tag{6}$$

MAE :

$$\mu_{i,j}^{k+1} = m_{i,j}^{k} + 2\lambda_{MAE}\, sgn(p(n,m) - x(n,m))\left(\frac{p(n,m) - x_{i,j}}{\sum_{i,j \in M^*} m_{i,j}^{k}}\right) \tag{7}$$

where λ_{MSE} and λ_{MAE} are convergence factors, and sgn the sign function. These two equations do not respect the constraint $m_{i,j} \in [0,1]$. So at each iteration, the minimum and maximum value of $m_{i,j}$ is searched, and all the coefficients are scaled by the linear transformation :

$$min_\mu = \min_{i,j \in M} \{\mu_{i,j}^{k+1}\} \tag{8}$$
$$max_\mu = \max_{i,j \in M} \{\mu_{i,j}^{k+1}\} \tag{9}$$
$$m_{i,j}^{k+1} = \frac{\mu_{i,j}^{k+1} - min_\mu}{max_\mu - min_\mu} \tag{10}$$

Then all the coefficients $m_{i,j}^{k+1}$ take values in the range 0-1, which guarantee the convergence of the algorithm.

3 QUADRATIC FILTERS

3.1 Second order Volterra filter

The output y(n,m) of a second order Volterra filter is :

$$y(n,m) = y_L(n,m) + \alpha y_Q(n,m) \tag{11}$$

where α is a constant and y_L, y_Q respectively are the linear and the quadratic component of the output, expressed as

$$y_L(n,m) = \sum_{i,j \in M} h_{i,j} x_{i,j} \tag{12}$$
$$y_Q(n,m) = \sum_{i,j \in M} \sum_{k,l \in M} w_{i,j,k,l}\, x_{i,j} x_{k,l} \tag{13}$$

Our operators are defined by the linear filter coefficients $h_{i,j}$, the quadratic filter coefficients $w_{i,j,k,l}$, and the scaling factor α. As observed in [4], to ensure the preservation to the output of a uniform luminance input, the following conditions must hold :

$$\sum_{i,j \in M} h_{i,j} = 1 \quad and \quad \sum_{i,j \in M} \sum_{k,l \in M} w_{i,j,k,l} = 0 \tag{14}$$

3.2 Gradient Like Enhancement

As mentionned in section 2, we propose filters whose coefficients depend on the set $\{m_{i,j}\}$ viewed as levels of confidence ($m_{i,j} \to 1$ for pixels similar to the current pixel value, respectively $m_{i,j} \to 0$ for the others). The first type of filter, called Gradient Like Enhancement (GLE) is defined by the following coefficients :

$$h_{i,j} = \frac{m_{i,j}}{\sum_{(i,j) \in M} m_{i,j}} \tag{15}$$
$$w_{i,j,k,l} = \begin{cases} m_{i,j} - \overline{m} & if \quad (k,l) = (0,0) \\ 0 & if \quad (k,l) \neq (0,0) \end{cases} \tag{16}$$
$$\alpha \qquad a\ predefined\ scaling\ factor \tag{17}$$

with $\overline{m} = \frac{1}{KL} \sum_{(i,j) \in M} m_{i,j}$ mean value of the set $(m_{i,j})_{(i,j) \in M}$.
$y_L(n,m)$ (linear low pass filter) is a weighted average on M, which takes into account the most likely pixel values,

and performs the noise reduction on the background region. To explain the gradient behaviour of the quadratic term, we write equation (13) as follows:

$$\begin{aligned} y_Q(n,m) &= x(n,m) \sum_{(i,j)\in M} (m_{i,j} - \overline{m}) x_{i,j} \\ &= x(n,m) \times \mathbf{G} \end{aligned} \tag{18}$$

where

$$\begin{aligned} \mathbf{G} &= \sum_{(i,j)\in M} (m_{i,j} - \overline{m}) x_{i,j} \\ &= \sum_{C_1} (m_{i,j} - \overline{m}) x_{i,j} + \sum_{C_2} (m_{i,j} - \overline{m}) x_{i,j} \\ &= \sum_{C_1} |m_{i,j} - \overline{m}| x_{i,j} - \sum_{C_2} |m_{i,j} - \overline{m}| x_{i,j} \end{aligned} \tag{19}$$

and

$$\begin{aligned} C_1 &= \{(i,j) \in M / m_{i,j} \geq \overline{m}\} \\ C_2 &= \{(i,j) \in M / m_{i,j} < \overline{m}\} \end{aligned} \tag{20}$$

$\mathbf{G}$ is a local difference between the pixels the most similar (C_1) to the current pixel and those less similar (C_2). It can be interpreted as a local estimate of the gradient. The multiplication by $x(n,m)$ yields a stronger enhancement for bright pixels which is in accordance with the Weber's law. In order to improve noise robustness, we can replace $x(n,m)$ by $y_L(n,m)$, and the filter equation becomes :

$$y(n,m) = y_L(n,m)[1 + \alpha \sum_{(i,j)\in M} (m_{i,j} - \overline{m}) x_{i,j}] \tag{21}$$

3.3 Laplacian Like Enhancement

In this section we propose an estimation of the local laplacian with the following quadratic coefficients :

$$w_{i,j,k,l} = \begin{cases} \sum_{M^*} |m_{i,j} - \overline{m}| & (i,j,k,l) = (0,0,0,0) \\ |m_{i,j} - \overline{m}| & \left\{ \begin{array}{l} (k,l) = (0,0) \\ (i,j) \neq (0,0) \end{array} \right. \\ 0 & otherwise \end{cases}$$

$y_L(n,m)$ and α are the same as in the previous section. The quadratic operator is then :

$$\begin{aligned} y_Q(n,m) = x(n,m)(&\sum_{(i,j)\in M^*} |m_{i,j} - \overline{m}| x(n,m) \\ &- \sum_{(i,j)\in M^*} |m_{i,j} - \overline{m}| x_{i,j}) \end{aligned} \tag{22}$$

Lets define $a_{i,j} = |m_{i,j} - \overline{m}|$, and consider a 3 by 3 mask M, the convolution mask of the difference is then comparable to a laplacian mask :

$$\begin{vmatrix} -a_{11} & -a_{12} & -a_{13} \\ -a_{21} & \sum a_{i,j} & -a_{23} \\ -a_{31} & -a_{32} & -a_{33} \end{vmatrix} \equiv \begin{vmatrix} -a & -a & -a \\ -a & 8a & -a \\ -a & -a & -a \end{vmatrix} \tag{23}$$

Thus the quadratic filter behaves like the local-mean-weighted highpass filter proposed in [2], and is then still in accordance with the Weber's law because of the multiplication by $x(m,n)$ (which could be replaced by $y_L(n,m)$ as explained in 3.2).

4 APPLICATION TO IMAGE ENHANCEMENT

The proposed filter scheme can be used to enhance blurred and noisy images. Results are first presented on the "canevas" image shown in figure 2. If we process such an image by the classical quadratic operators (figure 3.a: Ramponi's quadratic filter [4] with d=70, and figure 3.b: Mitra's quadratic filter [2] with $\alpha = 512$), we observe a noise amplification. Images obtained with GLE and LLE filters appear more convincing in terms of contrast enhancement (figures 3.c: GLE and LLE methods). The sharpening effect of three operators can be observed on the grey levels along a line of the output images as shown in figure 4. It should be observed that GLE yields better detail sharpening and noise smoothing.

The second application proposed is the iterative filtering of the "canevas" image corrupted by a zero-mean gaussian noise with standard deviation 60. Classicaly additive gaussian noise is removed by applying low pass filters, but at the expense of edge blurring. So it seems promising to process the image by a low pass filter to remove the noise, and a high pass filter to sharpen the edge avoiding noise amplification. We then have iteratively filtered the noisy image with $\alpha = 1000$, $\lambda = 0.01$ and $M = [5*5]$ (see fig. 5), and after four filtering we can observe the effective sharpening effect without noise amplification.

5 CONCLUSION

In this paper, two new quadratic filters for image enhancement have been presented. The novelty of our approach is the use of an adaptive filter mask in the quadratic part. The improvement introduced by our two filters has been shown through the filtering and enhancement of noisy and non noisy periodic textured images for which other recent quadratic filters failed.

References

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[5] G. Ramponi, "A simple cubic operator for sharpening an image", IEEE Workshop on NSIP, Thessalhoniki june 20-22 1995, pp 963-966.

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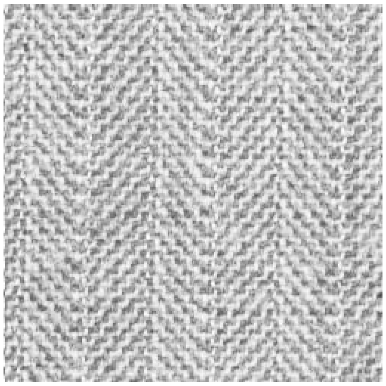
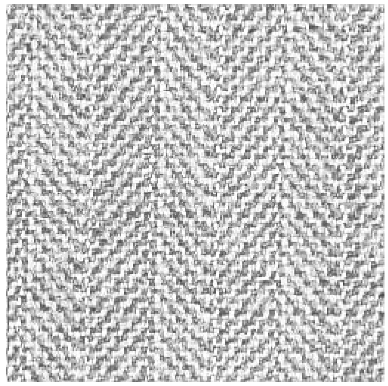
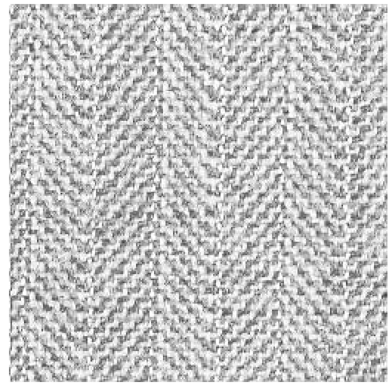


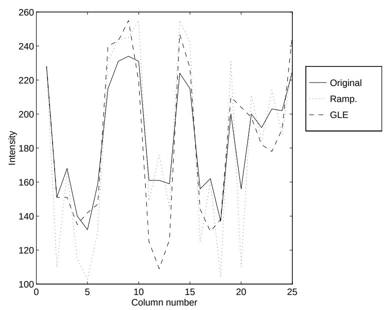
2 : initial image



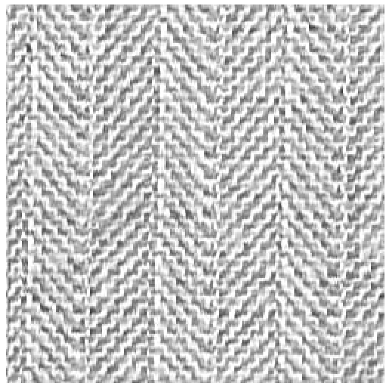
3.a



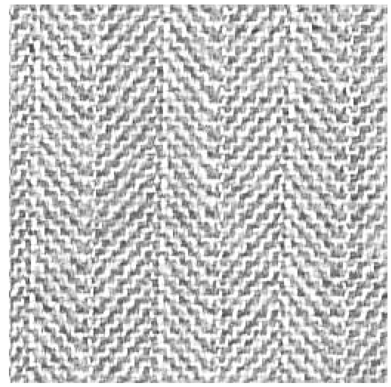
3.b



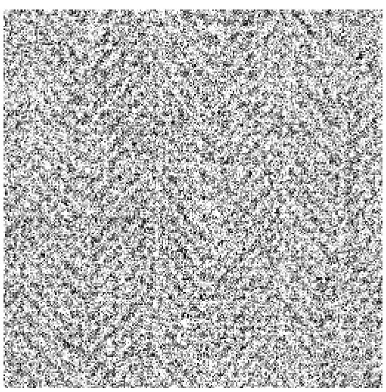
4



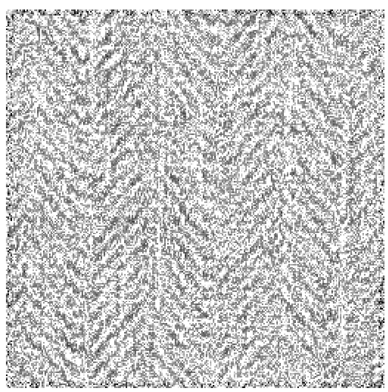
3.c 5*5 GLE $\alpha = 512$, $\lambda = 0.01$



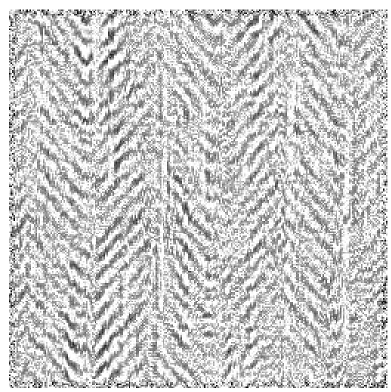
3.d 5*5 LLE $\alpha = 512$, $\lambda = 0.01$



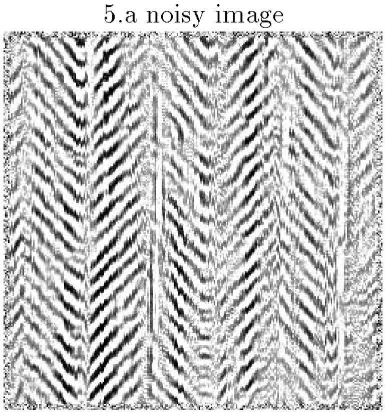
5.a noisy image



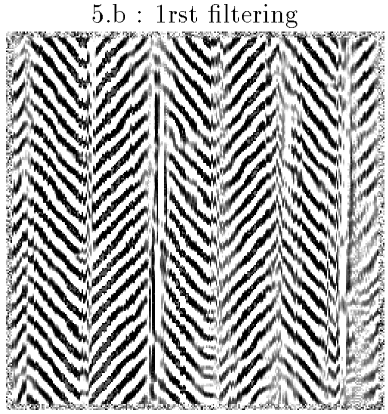
5.b : 1rst filtering



5.c : 2nd filtering



5.d 3rd filtering



5.e : 4th filtering