Evolutionary Design of Analog FIR Filters with Variable Time Delays for Optically Controlled Microwave Signal Processors

André Neubauer

Department of Communication Engineering
Duisburg Gerhard-Mercator-University
47048 Duisburg, Germany
Tel: +49 203 379 2842, Fax: +49 203 379 2902
E-mail: neubauer@sent5.uni-duisburg.de

ABSTRACT

This paper presents the application of genetic algorithms to the design of analog FIR filters with variable time delays — specific examples for tunable optically controlled microwave signal processors. Besides the FIR filter coefficients as the standard design parameters, the time delays can additionally be optimized. Because of physical constraints, specific restrictions of the design parameters, however, have to be obeyed. In order to make use of the additional freedom of optimizing the time delays and to observe the design restrictions, non-standard design techniques are needed. To this end, this paper studies the applicability of genetic algorithms to analog FIR filter design. Experimental results and a comparison to a standard design algorithm are given that demonstrate the excellent properties of the proposed evolutionary design technique.

1 Introduction

A promising approach to realize reconfigurable or tunable analog filters in the microwave frequency range up to 40 GHz is based on optically controlled microwave signal processors [6]. With the help of coplanar transmission lines on semiconducting substrate, *analog* nonrecursive filters with time-discrete impulse response — analog FIR (finite impulse response) filters — can be implemented. By optically controlling the coplanar transmission lines using a laser beam, the corresponding attenuation and phase factors can be varied, leading to variable filter coefficients and variable time delays for the FIR filter. Because of this specific hardware realization, the filter coefficients as well as the time delays can be optimized leading to a desired filter design. The restrictions are, however, non-negative filter coefficients with magnitude less than one and low filter order in order to obtain reasonable chip sizes. Standard techniques for the design of *digital* filters cannot readily account for the additional freedom of variable time delays as design parameters when dealing with analog FIR filters [10]. The incorporation of further restrictions with regard to the filter coefficients also poses problems. Therefore, *genetic algorithms* — powerful heuristic search procedures based on the mechanisms of molecular genetics and natural evolution — are applied to the design of analog FIR filters with variable time delays for optically controlled microwave signal processors. In contrast to recently proposed evolutionary design techniques for FIR digital filters [1, 2, 4, 11, 12, 13] or genetic algorithm based adaptive filters [7, 8, 9] the filter coefficients as well as the time delays are optimized.

Section 2 gives an introduction to the specific structure of analog FIR filters with variable time delays. The genetic algorithm that is applied to the FIR filter design is described in section 3 together with the definition of the fitness function. Experimental results for band-pass filter design and a comparison to the Parks-McClellan algorithm [10] with regard to the filter characteristics are then given in section 4. It is shown that the incorporation of the variable time delays as additional design parameters leads to improved filter designs even for low filter orders — a necessary requirement for the successful application of optically controlled microwave signal processors.

2 Analog FIR Filters with Variable Time Delays

The general structure of the analog FIR filter with variable time delays for optically controlled microwave signal processors is shown in figure 1. Given the input signal $s(t)$ with normalized time t, the output signal

$$g(t) = \sum_{i=1}^{n} a_i \cdot s(t - T_i) \tag{1}$$

is calculated with the help of the filter coefficients a_i and time delays T_i, $i = 1(1)n$. The analog FIR filter with variable time delays is stable in the BIBO (bounded input bounded output) sense since

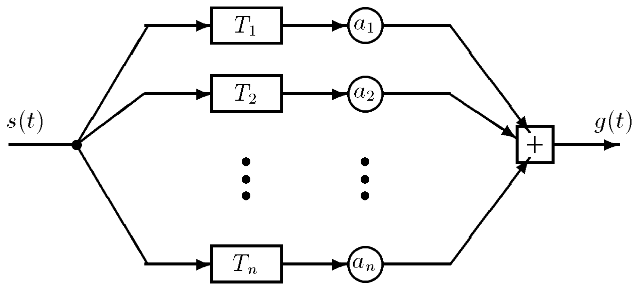

Figure 1: The structure of the analog FIR filter with variable time delays of order n.

$\max_{t\in\mathbf{R}}\{|g(t)|\} \leq \max_{t\in\mathbf{R}}\{|s(t)|\} \cdot \sum_{i=1}^{n} |a_i|$. The impulse response

$$h(t) = \sum_{i=1}^{n} a_i \cdot \delta(t - T_i) \qquad (2)$$

leads to the FIR filter's transfer function

$$H(f) = |H(f)| \cdot \mathrm{e}^{j\angle H(f)} = \sum_{i=1}^{n} a_i \cdot \mathrm{e}^{-j2\pi f T_i} \qquad (3)$$

with normalized frequency f.

For optically controlled microwave signal processors each branch of the FIR filter can be implemented with the help of SCHOTTKY contact coplanar microwave waveguides on semiconducting substrate [6]. Variable filter coefficients a_i and time delays T_i, $i = 1(1)n$, for the FIR filter of order n can be obtained by optically controlling the coplanar transmission lines with the help of a laser beam, thus varying the corresponding attenuation and phase factors. Because of this specific hardware realization, the filter coefficients a_i as well as the time delays T_i can be optimized in order to yield a desired filter design — given by a specified transfer function $H_d(f)$. The physical restrictions are, however, merely non-negative coefficients $0 \leq a_i \leq 1$. Since filter designs in the microwave frequency range ($f > 0$) are required, the condition $|H(f)| \leq H(0)$ that is implied by $a_i \geq 0$ is of no importance here. Further, the demand for small chip sizes leads to the required low order n.

Given a desired transfer function $H_d(f)$ — e.g. by specifying $|H_d(f)|$ and, if required, $\angle H_d(f)$ —, the design of analog FIR filters with variable time delays can be approached by optimizing the corresponding filter parameter vector

$$\vec{\theta} = (\theta_1, \ldots, \theta_N) = (a_1, \ldots, a_n, T_1, \ldots, T_n) \qquad (4)$$

with $N = 2n$ components according to a specific approximation or optimization criterion. Standard techniques for the design of *digital* FIR filters, however, merely optimize the filter coefficients a_i [10]. The time delays T_i are generally fixed by the sample rate of the processed discrete signals. Therefore, standard techniques for the design of *digital* FIR filters cannot readily account for the additional freedom of variable time delays T_i as design parameters when dealing with *analog* FIR filters. The incorporation of further restrictions with regard to the filter coefficients a_i also poses problems. Thus, the application of a special class of heuristic search procedures — genetic algorithms — to the design of analog FIR filters with variable time delays is proposed. In the next section, the structure of genetic algorithms is briefly discussed.

3 Genetic Algorithms

Genetic algorithms are powerful and robust heuristic search procedures taking pattern from molecular genetics and natural evolution [3, 5]. Here, they are applied to the optimization of the FIR filter's parameter vector $\vec{\theta}$ in equation 4. Genetic algorithms are based on a *population* $\psi(g)$ of *individuals* $\phi_h(g)$, $h = 1(1)M$, representing possible solutions for the problem under consideration at iteration step or *generation* g. Each individual $\phi_h(g) = (\varphi_{h,1}(g), \ldots, \varphi_{h,\ell}(g))$ is defined as an ℓ-tuple of binary *alleles* $\varphi_{h,k}(g) \in \{0, 1\}$, that represents one parameter vector $\vec{\theta}_h(g)$ of the analog FIR filter using a K-bit binary or GRAY code leading to $\ell = KN$. Especially, the design parameter $\theta_{h,i}(g)$, $i = 1(1)N$, is coded with the help of the alleles $\phi_{h,(i-1)\cdot K+1}(g), \ldots, \phi_{h,i\cdot K}(g)$. In order to create the new population $\psi(g+1)$ of generation $g+1$, a number of M new individuals is created. Based on the quality or *fitness* $F_h(g)$ of $\phi_h(g)$, two individuals $\phi_i(g)$ and $\phi_j(g)$ are selected from $\psi(g)$. Using *fitness proportionate selection* the selection probabilities are

$$p_h(g) = \frac{F_h(g)}{\sum_{h'=1}^{M} F_{h'}(g)} \qquad (5)$$

Two new individuals $\phi'_h(g)$ and $\phi'_{h+1}(g)$ are then generated with the help of genetic operators — *crossover* and *mutation*. The crossover-operator crosses the selected ℓ-tuples $\phi_i(g)$ and $\phi_j(g)$ with probability p_C at a randomly chosen site. Each allele of the resulting ℓ-tuples is then mutated with a small probability p_M, yielding $\phi'_h(g)$ and $\phi'_{h+1}(g)$. In figure 2, the structure of this simple genetic algorithm is given [3].

In order to produce a single result θ_i, $i = 1(1)N$, for each design parameter of the analog FIR filter with variable time delays, the parameters $\theta_{h^*,i}(g_{\max})$ of the best individual $\phi_{h^*}(g_{\max})$ in the final generation $g_{\max}$, i.e. $F_{h^*}(g_{\max}) = \max_{1\leq h\leq M} F_h(g_{\max})$, are chosen.

After having discussed the general structure of the genetic algorithm, the fitness function with regard to the analog FIR filter design is mathematically defined in the following. Given a desired characteristic for $|H_d(f)|$ and an FIR filter parameter vector $\vec{\theta}_h(g)$ the absolute

Initialize population $\psi(0)$ at random.

FOR $(g = 0;\ g \le g_{\max};\ g{+}{+})$

- Calculate fitness $F_h(g)$ of each individual $\phi_h(g)$, $h = 1(1)M$, in population $\psi(g)$ at generation g.
- FOR $(h = 1;\ h < M;\ h+=2)$
  - Select individuals $\phi_i(g)$ and $\phi_j(g)$, $1 \le i, j \le M$, from $\psi(g)$ according to the selection probabilities $p_h(g)$.
  - Apply crossover and mutation to $\phi_i(g)$, $\phi_j(g)$ to create offspring $\phi'_h(g)$, $\phi'_{h+1}(g)$.
  - Store $\phi'_h(g)$, $\phi'_{h+1}(g)$ in $\psi'(g)$.
- Set $\psi(g+1) = \psi'(g)$.

Figure 2: Genetic algorithm.

error criterion

$$E_h(g) = \frac{1}{N_f} \sum_{i=1}^{N_f} w(f_i) \cdot \big| |H_d(f_i)| - |H(\vec{\theta}_h(g), f_i)| \big| \tag{6}$$

is calculated with N_f frequency sample points f_i. $H(\vec{\theta}_h(g), f)$ denotes the transfer function of the analog FIR filter that is obtained for parameter vector $\vec{\theta}_h(g)$ of individual $\phi_h(g)$. The weight function $w(f)$ offers the possibility to emphasize specific frequency ranges that are of particular importance. The fitness for individual $\phi_h(g)$ is then given by

$$F_h(g) = \frac{1}{1 + E_h(g)} \tag{7}$$

4 Experimental Results

The genetic algorithm discussed above is applied to the design of band-pass filters in the normalized frequency range $[f_1, f_2]$. With the normalized lower and upper band-pass cut-off frequencies f_ℓ and f_u, respectively, the modulus of the desired transfer function $H_d(f)$ is given by

$$|H_d(f)| = \begin{cases} 0 & \text{if} \quad f_1 \le f < f_\ell \\ 1 & \text{if} \quad f_\ell \le f \le f_u \\ 0 & \text{if} \quad f_u < f \le f_2 \end{cases} \tag{8}$$

The weight function for the calculation of the quality criterion is

$$w(f) = \begin{cases} \beta_1 & \text{if} \quad f_1 \le f < f_\ell \\ \beta_2 & \text{if} \quad f_\ell \le f \le f_u \\ \beta_3 & \text{if} \quad f_u < f \le f_2 \end{cases} \tag{9}$$

As an example for the design of extremely small order filters ($n = 8$), the design of a band-pass filter with $f_\ell = 0.35$ and $f_u = 0.39$, respectively, is discussed in the frequency range $[f_1, f_2] = [0.25, 0.50]$. The weight function $w(f)$ with $\beta_1 = \beta_3 = 1$, $\beta_2 = 5$ and $N_f = 100$ equidistant frequency sample points in $[f_1, f_2]$ according to equation 6 are chosen. The parameter setting for the genetic algorithm is given in table 1. In order to incorporate the design restrictions $0 \le a_i \le 1$, the filter coefficients a_i are coded in the interval $[0, 1]$ with the help of a $K = 32$-bit binary code. Each time delay T_i is equivalently coded by $K = 32$ alleles in the interval $[0, 31]$.

crossover probability p_C	0.6
mutation probability p_M	0.001
population size M	100
number of generations $g_{\max}$	500

Table 1: The parameter setting for the genetic algorithm.

Figure 3 shows the impulse response $h(t)$ of the resulting analog FIR filter with variable time delays. In figure 4, the corresponding modulus of the transfer function $|H(f)|$ is given. The genetic algorithm leads to a filter characteristic that approximates the desired one.

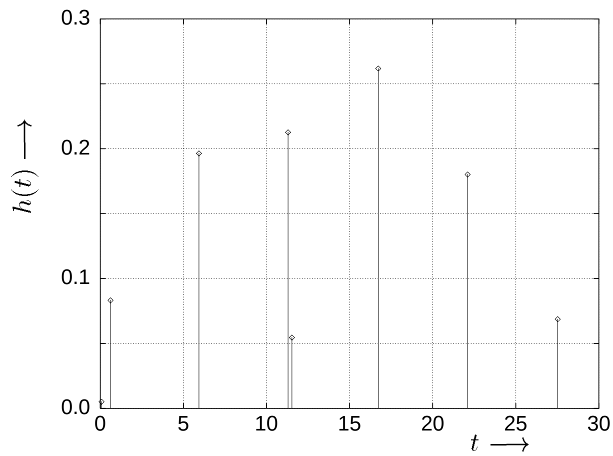

Figure 3: Impulse response $h(t)$ for the evolutionary design of 8th order FIR filter with variable time delays.

In order to value the proposed methodology, the results are compared with those obtained with the Parks-McClellan algorithm for the design of FIR filters with linear phase $\angle H(f)$ [10]. This algorithm optimizes the filter coefficients a_i while assuming equidistant time delays $T_i = i - 1$, $i = 1(1)n$. No design restrictions for the a_i ($0 \le a_i \le 1$) are taken into consideration. In figure 5, the corresponding modulus of the resulting transfer function $|H(f)|$ for a filter order $n = 16$ is given. The genetic algorithm leads to a superior FIR filter design with higher attenuation in the stop-bands. This is achieved despite the lower filter order n and the design restric-

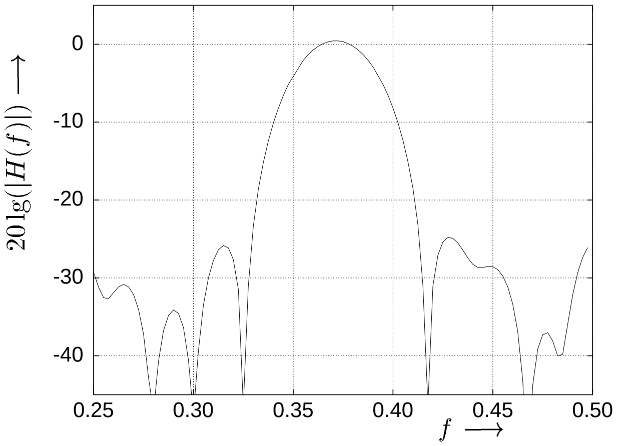

Figure 4: Transfer function $|H(f)|$ for the evolutionary design of 8th order FIR filter with variable time delays.

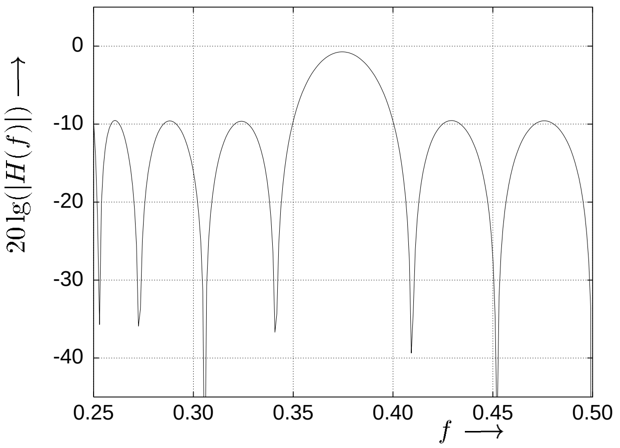

Figure 5: Transfer function $|H(f)|$ for the Parks-McClellan design of 16th order FIR filter.

tions $0 \leq a_i \leq 1$ considered in the evolutionary design. Further, the incorporation of variable time delays as additional design parameters leads to an improvement of the filter characteristics.

5 Conclusions and Outlook

The application of genetic algorithms to the design of analog FIR filters with variable time delays for optically controlled microwave signal processors was proposed. The additional freedom of variable time delays as well as the restrictions with regard to the filter coefficients were easily implemented in the genetic algorithm. Experimental results for evolutionary band-pass filter design and a comparison to the standard Parks-McClellan algorithm demonstrated the excellent properties of the evolutionary design technique even for low filter orders. Further studies will extend the proposed evolutionary design methodology to recursive analog IIR filters with variable time delays.

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