EFFICIENT ALLOCATION OF POWER-OF-TWO TERMS IN COMPLEX FIR FILTER DESIGN

Tolga Çiloglu* and Yong Hoon Lee**
*Dept. of Electrical and Electronics Eng., Middle East Tech. Univ., Ankara, 06531, Turkey
e-mail: ciltolga@rorqual.cc.metu.edu.tr
**Dept. of Electrical Eng., Korea Advanced Institute of Science and Technology, Taejon, Korea
e-mail: yohlee@eekaist.kaist.ac.kr

Abstract

The design of discrete coefficient FIR filters with arbitrary magnitude and phase specifictions and whose coefficients are expressed as the signed combination of a few power-of-terms (SPT) is considered. The total number of SPT terms is fixed and their distribution among the coefficients is not restricted. The proposed method is an improved version of those originally proposed for the design of linear phase filters [9], [10].

1 INTRODUCTION

The design of digital filters, especially linear phase FIR filters with discrete valued coefficients has been an attractive research field basically because of the requirement for multiplierless implementation which provides high speed processing and reduces the chip complexity. Over the years the research in this area has been focussed on the design methods for the so called integer coefficient filters[1] [1]-[3] and the methods where a fixed number of signed power-of-two (SPT) terms are reserved for each filter coefficient [4]-[7]. Later, a number of algorithms which does not restrict a priori the number of SPT terms per coefficient are introduced [7]-[11]. In [8] an upper limit on the total number of SPT terms is assumed and the SPT terms are sequentially allocated to the filter coefficients, $\bar{h}$, to minimize

$$\max_n \left| \left[\bar{h} - \bar{h}_{ip} \right]_n \right|$$

where $[\ ]_n$ denotes the nth element of the vector inside the brackets and $\bar{h}_{ip}$ is the optimal infinite precision coefficient vector. This method compares favorably with the previous ones and it also has low computational complexity. A modification of this algorithm in [9] provided some more improved results.

The design of discrete coefficient FIR filters with arbitrary magnitude and phase specifications has been reported in [12]. An iterative weighted least squares algorithm is used as the core of the method. It is shown that highly improved results can be obtained with at most two SPT terms for each coefficient compared to those of simple rounding. In general the major drawback of such an approach is due to the lack of a unique method to adjust the frequency weighting function. Since there is no characterization of the frequency response of a discrete coefficient filter in the chebychev sense, the adjustment strategy has to involve heuristics.

In this paper the design of FIR filters with arbitrary magnitude and phase specifications in the chebychev sense by a variation of the method which proved to be very fruitful in the design of linear phase FIR filters [ýasted] is described. No restriction is imposed on the distribution of SPT terms among the filter coefficients. Essentially this is a post optimization method starting from the initial point obtained by the method of [8]. The infinite precision optimal coefficients are obtained by linear programming. It is shown through design examples that highly improved solutions over those of [12] can be obtained via the proposed method.

2 THE PROBLEM SETTING

Consider the transfer function of an FIR filter

$$H(z) = \sum_{n=0}^{N} h(n)\, z^{-n}$$

[1] The coefficients takes values from the set with elements of uniform spacing 2^{-b+1}, where b is the wordlength

where the coefficients h(n) are to be determined to minimize the cost function

$$C(\vec{h}) = \max_{\omega} \left| W(e^{j\omega}) \left[\frac{1}{b} H(e^{j\omega}) - D(e^{j\omega}) \right] \right|$$

where W(·) is the weighting function. The value of the scale factor *b* is determined prior to optimization and it deeply affects the final performance. The coefficients are expressed as

$$h(n) = \sum_{i=1}^{B_N} s_n(i)\, 2^{-p_n(i)}$$

where

$$s_n(i) \in \{-1,0,1\}, \quad p_n(i) \in \{1,2,\ldots,E\}$$

and

$$\sum_{k=0}^{N} B_k = P$$

When the total number, P, of SPT terms is fixed without restricting their distribution over the filter coefficients, the resulting discrete coefficient domain, Ψ, does not allow the discretization of filter coefficients independently. As a consequence, most of the previously developed approaches are not suitable for this case. Even simple rounding is not a trivial task and the method in [8] indeed finds the rounded solution and no frequency domain criterion is employed during the design process. The modification of this algorithm in [9] incorporates a frequency domain criterion in the allocation of SPT terms. The goal is to provide a more consistent decrease in the cost function as the new terms are assigned to the filter coefficients. However the intermediate improvements obtained due to relatively former allocations do not necessarily accumulate for the benefit of the final solution. This observation led us to perform a powerful search for the allocation of the final few SPT terms.

3 THE METHOD

Assuming that a total of P SPT terms are available for representing the filter coefficients, the algorithm of [8] is used to find the initial solution with P-D terms as the starting point, where D is a small positive integer. Then elaborate search is performed for the allocation of the remaining D SPT terms.

The algorithm systematically forms all L-tuples, L=1,2,...,D, of coefficients and for each L-tuple, all possible ways of allocating L to D SPT terms are considered. For example, if D=4 and L=3, there are four ways of allocating the SPT terms to the coefficient triples: 1-1-1, 1-1-2, 1-2-1, 2-1-1,. Note that although the number of totally spent SPT terms is one less in the first case, it may be the one yielding the best result.

The serach will be described based on the following notation: Let s_k , k=1,...,L denote the initial number of SPT terms of the k^{th} element of the L-tuple and s^+_k, k=1,...,L denote the number of SPT terms to be allocated to that coefficient then Γ_k denotes the discrete set which contains all combinations of up to $s_k+s^+_k$ SPT terms. Once an L-tuple of coefficients is picked the search is performed as follows: L-1 coefficients in that L-tuple are allowed to have values in a ±Δ neighbourhood around their initial values in the corresponding Γ_k's and for each fixed L-1 coefficients the remaining coefficient is varied to minimize the error. To reduce the number of computations, the correct direction of change is found by evaluating the derivative of the error with respect to that coefficient.

The combined effect of searching in a ±Δ neighborhood and and the way the sets Γ_k's are defined yields an algorithm which is capable of making adjustments on the rest of the coefficients while allocating new SPT terms to some of them.

4 EXAMPLES

In this section the results of 16 design examples are presented. The filter specifications are the same as those in [12] and the comparison of the results is provided. The number of samples in evaluating the cost function over the normalized frequency range [0,0.5] is set to 8 times the filter length. In the representation of coefficients, the number *E* is taken as 12 hence the resolution is the same as in [12]. The weighting function is uniform over all bands of interest. The filters are low-pass with odd lengths varying from 11 to 41. The normalized passband and stopband edges are [0,0.15] and [0.25,0.5], respectively. The desired constant group delay is 8 in all cases. The total number of SPT terms is twice the number of filter coefficients. The results are shown in Figure 1. The results of [12] are also shown on the same plot. It should be noted that the improvement obtained in this work is mainly due to the unrestricted distribution of SPT terms among the filter coefficients. The maximum number of SPT terms allowed for a coefficient in [12] is two.

References

[1] D. M. Kodek, ``Design of Optimal Finite Wordlength FIR Digital Filters using Integer Programming Techniques", *IEEE Trans. on ASSP*, vol.28, pp.304-308, June, 1980.
[2] D. M. Kodek, K. Steiglitz, ``Comparison of Optimal and Local Search Methods for Designing Finite Wordlength FIR Digital Filters'', IEEE Trans. on CAS., vol. 28, pp. 28-32, Jan. 1981.
[3] Y. C. Lim, S. R. Park, ''Discrete coefficents FIR Digital Filter Design Based pon an LMS Criteria'', IEEE Trans. CAS, vol. 30, pp. 723-739, Oct. 1983.
[4]N. Benvenuto et. al., ``Applications of Simulated Annealing for the design of Special Digital Filters", *IEEE Trans. on SP*, vol.40, pp. 323-332, Feb. 1992.
[5] Y. C. Lim,``Design of Discrete-Coefficient-Value Linear PhaseFIR Filters with Optimum Normalized Peak Ripple Magnitude", *IEEE Trans. CAS*, vol.37, pp. 1480-1486, December, 1990.
[6] Q. Zhao and Y. Tadokoro,``A Simple Design of FIR Filterswith Powers-of-Two Coefficients ", *IEEE Trans. on CAS*, vol.35, pp. 566-570, May, 1988.
[7] H. Samueli, ``An Improved Search Algorithm for the Design of Multiplierless FIR Filters with Powers-of-Two Coefficients", *IEEE Trans. on CAS*, vol.36, pp. 1044-1047, July, 1989.
[8] D. Li, J. Song and Y. C. Lim, ``A Polynomial-Time Algorithm for Designing Digital Filters with Power-of-Two Coefficients", *Proc. of ISCAS*, vol.1, pp. 84-87, 1993.
[9] C-L. Chen, K-Y, Khoo, A. N. Willson, Jr., ``An Improved Polynomial-Time Algorithm for Designing Filters with Power-of-Two Coefficients", *Proc. of ISCAS*, vol.1, pp. 223-226, 1995.
[10] H. Shaffeu et. al., ``Improved Design Procedure for Multiplierless FIR Digital Filters", *Elect. Letters*, vol.27, pp.1142-1144, June, 1991.
[11] D. Ait-Boudaoud and R. Cemes, ``Modified Sensitivity Criterion for the Design of Powers-of-Two FIR Filters", *Electronics Letters*, vol.29, pp. 1467-1469, Aug., 1993.
[12] J-H. Lee, C-K. Chen, Y-C. Lim, ``Design of Discrete Coefficient FIR Digital Filters with Arbitrary Amplitude and Phase Responses'', IEEE Trans. on CAS II., vol. 40, pp. 444-448, July. 1993.

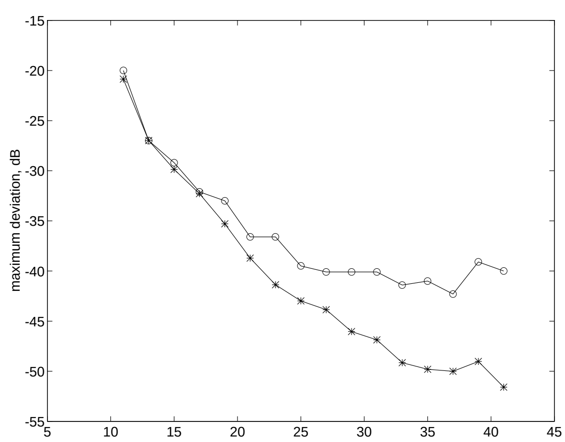

Figure 1. The plot of filter length vs. maximum deviation for the design examples. o The results of [12]
* The results of this work