FLEXIBLE NONUNIFORM FILTER BANKS USING ALLPASS TRANSFORMATION OF MULTIPLE ORDER

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ABSTRACT

This paper deals with allpass frequency transformations of uniform filter banks to achieve nonuniform bandwidths. The known transformation with an allpass of first order [1] [4] [5] [6] is extended to an allpass transformation of order K. Thus the flexibility of the filter bank design can be increased significantly.

1 INTRODUCTION

In this contribution the polyphase realization of the uniform FIR filter bank is generalized towards a nonuniform frequency resolution. The proposed approach is an extension of the well known allpass transformation of first order [1] [4] [5] by introducing an allpass of order K. The frequency transformation of the filter characteristics is achieved by replacing the delay elements of the FIR filter bank by identical recursive allpasses. A causal allpass of order K would create a filter bank with multiple images of the original (bandpass) filters, which can of course be useful e.g. for the design of comb filters [3]. Here a new phase compensation technique is introduced which avoids the multiple mapping of the frequency axis and therefore gives an increased design flexibility by using an allpass of order K.

2 UNIFORM FILTER BANK

Filter banks with uniform frequency resolution can be implemented very efficiently using a polyphase network (PPN) and the Fast Fourier Transformation (e.g. [5]). There are two equivalent versions which can be described by using either complex modulators with uniformly spaced frequencies and identical lowpasses or modulated bandpass filters which have been derived from a common prototype lowpass. Here, the latter version will be considered. If the impulse response of the FIR prototype lowpass is denoted by $w_0(n)$, the complex bandpass impulse responses with the center frequencies $\Omega_\mu = 2\pi\mu/N \quad (\mu = 0 \ldots N-1)$ are given by

$$w_\mu(n) = w_0(n) e^{+j\frac{2\pi}{N}\mu n} \qquad \mu = 0 \ldots N-1 \tag{1}$$

The subband signal of the μ'th channel of the uniform filter bank can be described by

$$y_\mu(n) = x(n) * w_\mu(n) \tag{2}$$

where x is the input signal and $*$ denotes convolution (see fig. 1).

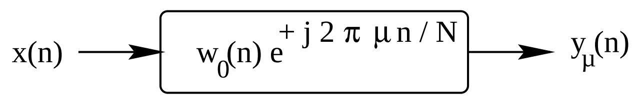

Figure 1: Reference model of the μ'th channel of a uniform bandpass filter bank

For simplicity it is assumed that an N-tap FIR prototype lowpass $w_0(n)$ is used[1]. In this case (eq. 2) reads

$$y_\mu(n) = \sum_{\nu=0}^{N-1} \underbrace{x(n-\nu) w_0(\nu)}_{x_\nu(n)} e^{+j\frac{2\pi}{N}\mu v} \tag{3a}$$

$$= \sum_{\nu=0}^{N-1} x_\nu(n) e^{+j\frac{2\pi}{N}\mu v} \tag{3b}$$

$$= \text{IDFT}\{x_\nu(n)\} \tag{3c}$$

Thus the set of samples $y_\mu(n) \quad$ (n=fixed, $\mu = 0 \ldots N-1$) can be calculated efficiently using the Inverse Fast Fourier Transformation (IFFT). If we introduce the z–domain version of $x_\nu(n)$ according to

$$X_\nu(z) = X(z) z^{-\nu} w_0(\nu) \tag{4}$$

we obtain the subband signals in the z–domain

$$Y_\mu(z) = \sum_{\nu=0}^{N-1} X_\nu(z) e^{+j\frac{2\pi}{N}\mu v} \tag{5}$$

as shown in fig. 2:

[1] This approach can easily be extented to prototype filters with more than N taps.

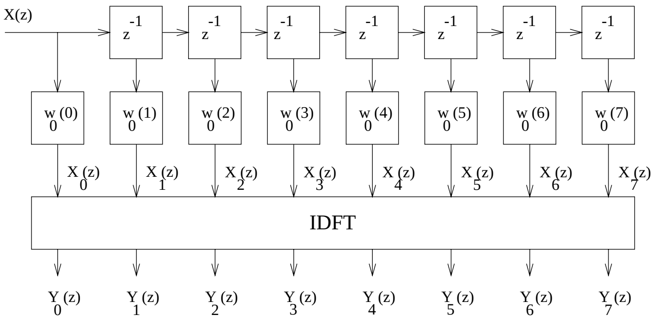

Figure 2: IFFT implementation of a uniform filter bank (N=8)

The effective frequency responses are:

$$H_\mu(z) = Y_\mu(z)/X(z) = \sum_{\nu=0}^{N-1} w_0(\nu) z^{-\nu} e^{+j\frac{2\pi}{N}\mu\nu} \tag{6}$$

On the unit circle i.e. for $z = e^{j\Omega}$ we obtain the following transfer functions

$$H_0(e^{j\Omega}) = \sum_{\nu=0}^{N-1} w_0(\nu) e^{-j\nu\Omega} \quad \text{(prototype filter)} \tag{7}$$

$$H_\mu(e^{j\Omega}) = H_0(e^{j(\Omega - \frac{2\pi}{N}\mu)}) \tag{8}$$

The frequency responses of this uniform prototype filter bank given by (eq. 8) are shown in fig. 3 by example.

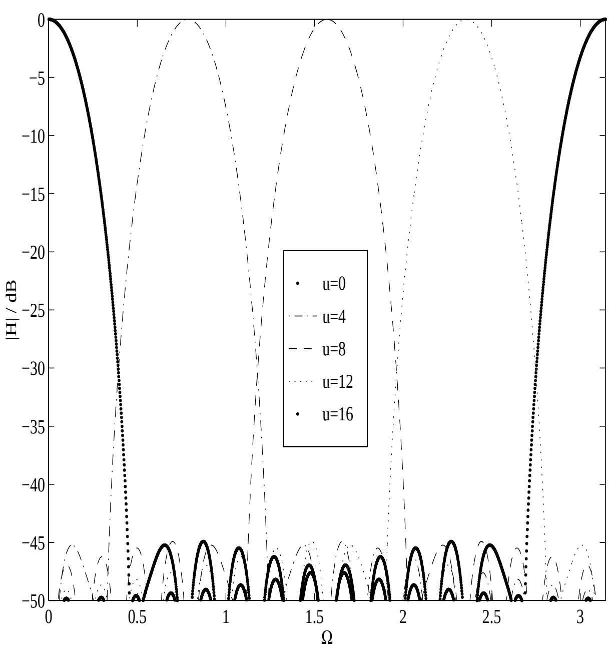

Figure 3: Uniform filter bank with allpass transformation of order $K = 1, a = 0$ (delay), $N = 32, \mu = 0, 4, 8, 12, 16$

3 GENERALIZED FILTER BANK

The filter structure of fig. 2 is generalized as shown in fig. 4 by replacing the delay elements by identical allpass filters $A(z)$ and by replacing the constant weights $w_0(\nu)$ by $B_\nu(z)$ $\quad (\nu = 0 \ldots N-1)$.

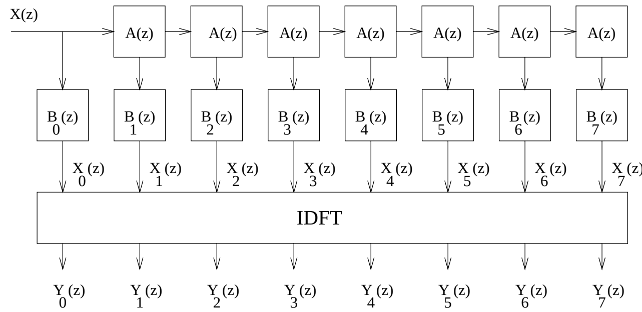

Figure 4: Generalized filter bank using IDFT ($N = 8$)

The generalized IDFT (IFFT) filter bank structure is described by:

$$Y_\mu(z) = \sum_{\nu=0}^{N-1} X_\nu(z) e^{+j\frac{2\pi}{N}\mu\nu} \tag{9}$$

$$= \sum_{\nu=0}^{N-1} X(z) B_\nu(z) \left[A(z)\right]^\nu e^{+j\frac{2\pi}{N}\mu\nu} \tag{10}$$

The N outputs ($\mu = 0 \ldots N-1$) of the filter bank in fig. 4 are now characterized by their transfer functions:

$$H_\mu(z) = \frac{Y_\mu(z)}{X(z)} = \sum_{\nu=0}^{N-1} B_\nu(z) \left[A(z)\right]^\nu e^{+j\frac{2\pi}{N}\mu\nu} \tag{11}$$

3.1 Uniform Filter Bank As Special Case

A special case of (eq. 11) is obviously the well known uniform polyphase filter bank (eq. 8) with the prototype filter (eq. 7) if we choose

$$A(z) = z^{-1} \qquad B_\nu(z) = w_0(\nu) \tag{12}$$

3.2 Allpass Transformation Of First Order

The substitution of delay elements z^{-1} in (eq. 12) by the causal and stable allpass of order $K = 1$ with the complex parameter $\underline{a} = a e^{j\alpha}$ with $|\underline{a}| < 1$ leads to:

$$A(z) = A_1(z) = \frac{1 - \underline{a}^* z}{z - \underline{a}} \qquad B_\nu(z) = w_0(\nu) \tag{13}$$

with

$$A_1(e^{+j\Omega}) = e^{+j\varphi_1(\Omega)} \quad \text{and} \tag{14}$$

$$\varphi_1(\Omega) = -\Omega - 2\arctan\frac{a\sin(\Omega - \alpha)}{1 - a\cos(\Omega - \alpha)} \tag{15}$$

which is the known nonuniform frequency transformation of the uniform filter bank of (eq. 8), describing the frequency responses:

$$\begin{aligned}\widetilde{H}_\mu(e^{j\Omega}) &= \sum_{\nu=0}^{N-1} w_0(\nu) e^{+j\frac{2\pi}{N}\mu\nu} e^{+j\nu\varphi_1(\Omega)} \\ &= H_\mu(e^{-j\varphi_1(\Omega)}) \\ &= H_0(e^{+j(-\varphi_1(\Omega)-\frac{2\pi}{N}\mu)}) \end{aligned} \quad (16)$$

Generally speaking, the negative phase characteristic of the allpass describes the frequency transformation of the uniform prototype filter bank. In fig. 5 the negative phase of an allpass of first order ($a = 0.5$) and in fig. 6 the corresponding nonuniform filter bank transfer functions are plotted.

3.3 Allpass Transformation Of Higher Order With Multiple Mapping

The substitution of delay elements z^{-1} in (eq. 12) by a causal and stable allpass of order K with the complex parameters $\underline{a}_k = a_k e^{j\alpha_k}$ with $|\underline{a}_k| < 1, \quad k = 1 \dots K$ and the prototype filter of (eq. 7) leads to:

$$A(z) = A_K(z) = e^{j\alpha_0} \prod_{k=1}^{K} \frac{1 - a_k e^{-j\alpha_k} z}{z - a_k e^{j\alpha_k}} \quad (17)$$

$$A_K(e^{j\Omega}) = e^{+j\varphi_K(\Omega)} \qquad B_\nu(z) = w_0(\nu)$$

with the phase of this allpass:

$$\varphi_K(\Omega) = \alpha_0 - K\Omega - 2\sum_{k=1}^{K} \arctan \frac{a_k \sin(\Omega - \alpha_k)}{1 - a_k \cos(\Omega - \alpha_k)} \quad (18)$$

It can be shown that the phase φ_K (eq. 18) of an **causal** stable allpass of order K of $A_K(z)$ (eq. 17) is monotone decreasing [2]. Thus the (2π periodic) frequency interval $[-K\pi; K\pi]$ of the prototype filter is mapped to the interval $[-\pi; \pi]$. This is equivalent to multiple compressed mapping of $[-\pi; \pi]$ of the prototype filter to the target range $[-\pi; \pi]$ as proposed in [3] for the design of comb filters.

3.4 Allpass Transformation Of Higher Order With Single Mapping

In order to exploit the enhanced flexibility of the allpass of order K but to avoid multiple mappings, the phase has to be limited to $[-\pi; \pi]$. This can be achieved by reducing the linear term $K\Omega$ in (eq. 18) to Ω, resulting in a **non causal** stable allpass $\widehat{A}_K(z)$ (eq. 20) with a phase $\widehat{\varphi}_K$ (eq. 19):

$$\widehat{\varphi}_K(\Omega) = \varphi_K(\Omega) + (K-1)\Omega \quad (19)$$

$$\widehat{A}_K(z) = z^{+(K-1)} A_K(z) \quad (20)$$

To avoid frequency reversions in the resulting filter structure, the phase $\widehat{\varphi}_K(\Omega)$ of the **non causal** allpass has to be monotone decreasing which leads to the group delay constraint (eq. 21) of the allpass $A_K(z)$ (eq. 17).

$$-\frac{d}{d\Omega}\varphi_K(\Omega) = \sum_{k=1}^{K} \frac{1 - a_k^2}{1 - 2a_k\cos(\Omega - \alpha_k) + a_k^2} \overset{!}{>} K - 1 \quad (21)$$

An example of allpass parameters which achieve a decreasing phase $\widehat{\varphi}_K(\Omega)$ is shown for $K = 2$ in fig. 5 in comparison to the first order transformation characteristic $\varphi_1(\Omega)$. This demonstrates the enhanced flexibility in designing the nonuniform frequency transformation. The allpass of order $K = 2$ with conjugated complex parameters can be implemented by a network with real valued arithmetic.

3.5 Implementation

With (eq. 11) there are **non causal** implementations which have the same transfer functions:

$$A(z) = \underbrace{\widehat{A}_K(z)}_{\text{non causal}} \qquad B_\nu(z) = w_0(\nu) \quad (22a)$$

$$A(z) = \underbrace{A_K(z)}_{\text{causal}} \qquad B_\nu(z) = \underbrace{z^{+(K-1)\nu} w_0(\nu)}_{\text{non causal}} \quad (22b)$$

$$\nu = 0, 1 \dots N-1$$

If the **non causal** part is shifted to $B_\nu(z)$ according to (eq. 22b), a **causal** implementation is possible by introducing an additional delay of $z^{-(K-1)(N-1)}$ resulting in:

$$A(z) = A_K(z) \qquad \nu = 0, 1 \dots N-1$$

$$B_\nu(z) = z^{-(K-1)(N-1-\nu)} w_0(\nu) \quad (23)$$

The frequency response of this transformed **causal** transformed filter bank now reads:

$$\begin{aligned}\widehat{H}_\mu(e^{j\Omega}) &= e^{-j(N-1)(K-1)\Omega} \\ &\cdot \sum_{\nu=0}^{N-1} w_0(\nu) e^{+j\frac{2\pi}{N}\mu\nu} e^{+j\nu\widehat{\varphi}_K(\Omega)} \\ &= e^{-j(N-1)(K-1)\Omega} H_\mu(e^{-j\widehat{\varphi}_K(\Omega)}) \\ &= e^{-j(N-1)(K-1)\Omega} H_0(e^{+j(-\widehat{\varphi}_K(\Omega)-\frac{2\pi}{N}\mu)}) \end{aligned} \quad (24)$$

The block diagram is given in fig. 4

4 EXAMPLE AND CONCLUSIONS

The frequency responses of the compensated nonuniform filter bank for e.g. $K = 2$ are shown in fig. 7 revealing the enhanced flexibility of the new approach. In comparison to the transformation of order $K = 1$ as shown in fig. 6 the frequency resolution can now be increased e.g. within a bandpass interval. Thus the filter structure allows more parameters to design nonuniform filter banks using a common prototype FIR filter.

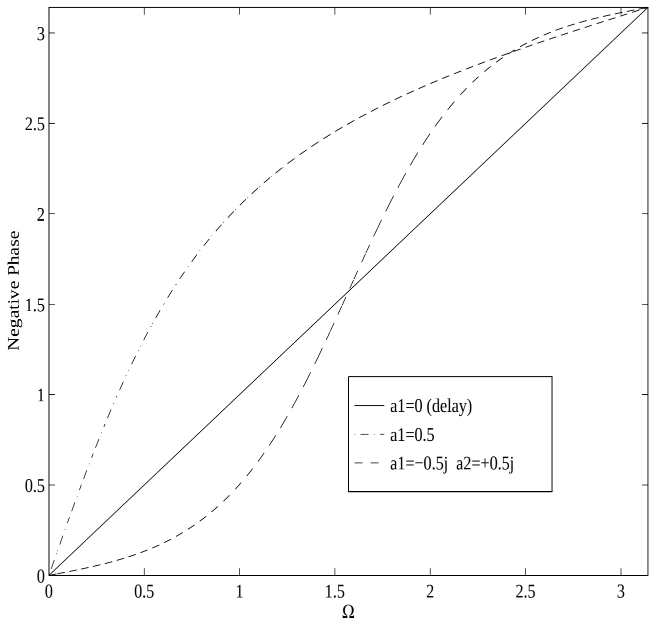

Figure 5: Frequency transformation with allpass of order $K = 1$ $(a1 = 0, a1 = 0.5)$ and allpass with order $K = 2$ with conjugate complex coefficients $(a1 = -0.5j,\ \ a2 = +0.5j)$ with compensation of the non causal phase

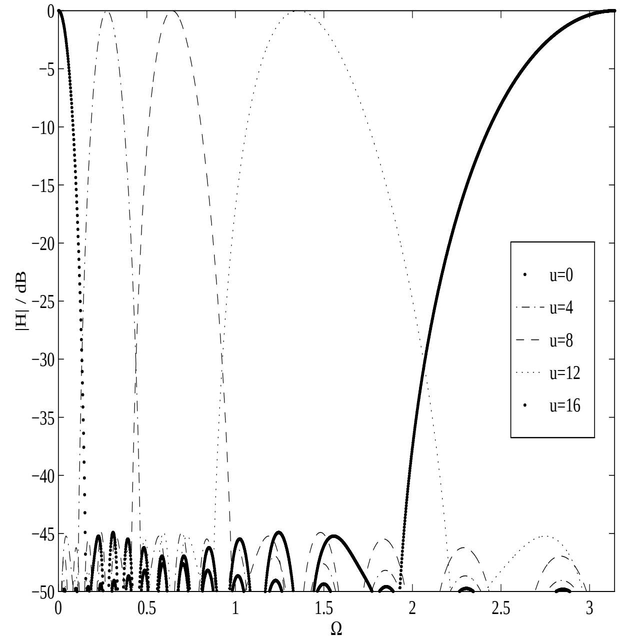

Figure 6: Nonuniform filter bank with allpass transformation of order $K = 1, a = 0.5, N = 32, \mu = 0, 4, 8, 12, 16$

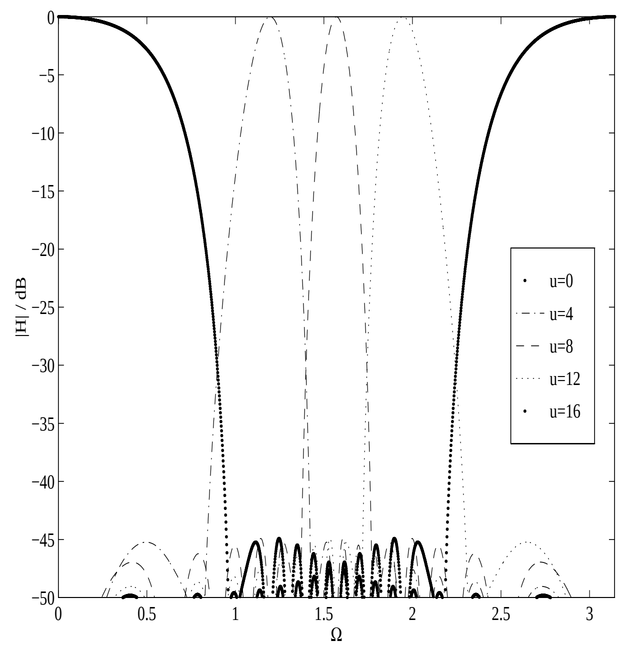

Figure 7: Nonuniform filter bank with allpass transformation of order $K = 2$ and compensation of the non causal phase $(a1 = -0.5j, a2 = +0.5j), N = 32, \mu = 0, 4, 8, 12, 16$

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