A SUPER-RESOLUTION METHOD BASED ON THE DISCRETE COSINE TRANSFORMS

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ABSTRACT

A super-resolution method based on the discrete cosine transform (DCT) is proposed for a signal with some frequency damage under a type 1 linear-phase (LP) FIR filter as a damage model. The proposed method can be carried out with real value operation and is applicable to any DCT in 4 kinds of DCTs. In addtion, two magnification schemes based on the proposed method to improve the conventional scheme are described.

1 INTRODUCTION

Methods for estimating some frequency components damaged due to sampling or other processes are referred to as super-resolution methods in the frequency domain. Some super-resolution methods such as the Gerchberg-Papoulis (GP) iterative method[1][2] have been explored[1]-[6]. These methods, sometimes called the DFT-based schemes, are developed as techniques for solving simultaneous linear equations. These equations are formulated from the relationship between signals in the time domain and their discrete fourier transforms (DFTs). From this point of view, some convergence conditions for the super-resolution methods are also shown[6][7].

However, it is necessary to use complex values even when signals are real. To avoid this problem, the GP-iterative method with the discrete cosine transform (DCT) was applied to the image magnification[8]. Although this method has real value operation, the convergence of the error between the original signal and the estimated signal is not guaranteed[8].

In this paper, we propose a new super-resolution method based on the DCT. Firstly, we investigate the DCT of a signal damaged in the high frequency components for the formulation of a super-resolution method in the DCT domain for a signal with some frequency damage. Then, we show that, if the damage process can be modeled as linear convolution with a type 1 linear phase (LP) FIR filter, some DCT coefficients of a damaged signal are the same as those of the original signal except for the DCT coefficients corresponding to the high frequency components. From this investigation, a new super-resolution method based on the DCT is proposed. This method can be regarded as expanding the DFT-based scheme[6]. It is also suitable for four type of the DCTs. Besides, we expand the proposed method to two magnification schemes to obtain interpolated signals. Finally, we show some computer simulations to verify the significance of the proposed method.

2 A NEW SUPER-RESOLUTION METHOD

In this section, we will propose a new super-resolution method based on the DCT, referred to as the DCT-based scheme, for a signal with some frequency damage. If the damage process can be modeled as linear convolution with a type 1 LP FIR filter, the proposed super-resolution method is provided for the DCTs with four types. An LP FIR filter as a damage model means that damaged signals do not include phase distortion. Moreover, the condition of type 1 allows us to do a super-resolution method based on the relationship between time and DCT domains for a signal with some frequency damage.

In the following, we will explain the damage process assumed in this paper. Then, the proposed DCT-based scheme will be described.

2.1 A Model of the Damage Process

First, we will explain a constraint for the damage process. In order to formulate a super-resolution method based on the DCT for a signal damaged in the high frequency components, it is necessary that some DCT coefficients of the damaged signal are the same as those of the original one except for the DCT coefficients corresponding to the high frequency components. If the damage process can be modeled as linear convolution with a type 1 LP FIR filter, the above condition will be satisfied. Firstly, note that the following properties provided in the article[10].

1. The DCT coefficients of a finite sequence $x(n)$ can be obtained by calculating the generalized discrete Fourier transform[10] of a symmetric-periodic sequence (SPS) $\hat{x}(n)$ extended from $x(n)$.

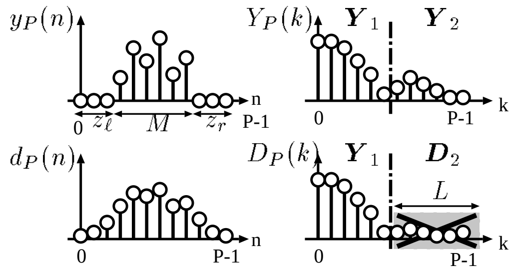

Figure 1: The original signal $y_P(n)$ and the observed signal $d_P(n)$.

2. The linear convolution with an LP FIR filter holds both of the symmetric and periodic properties of SPS and type 1 LP FIR filters especially hold the type of the symmetric property.

Based on the above two properties, we can show that, if the damage process can be modeled as linear convolution with a type 1 LP FIR filter, some DCT coefficients of the observed signal damaged in high frequency components are the same as those of the original one except for the DCT coefficients corresponding to the high frequency components[9]. Thus, we can formulate the DCT-based scheme for a signal damaged in the high frequency components.

2.2 The Proposed Method

In the following, we propose a new super-resolution method under two assumptions for executing the DCT-based scheme. The proposed DCT-based scheme can be formulated as a technique for solving simultaneous linear equations. For convenience of the discussion, we consider only type-II DCT (DCT-II).

2.2.1 Assumptions

As shown in Fig. 1, M points of the P-point original signal $y_P(n)$ are not guaranteed to have zero values. The observed signal $d_P(n)$ is damaged in some DCT coefficients corresponding to the high frequency components, where $Y_P(k)$ and $D_P(k)$ are the DCT coefficients of $y_P(n)$ and $d_P(n)$, respectively. Now, let L be the number of the damaged DCT coefficients, $Y_P(k) \neq D_P(k)$. The proposed DCT-based scheme is executed under two assumptions as follows:

(1) We know the time interval that is not guaranteed $y_P(n)$=0 and its number M.

(2) The DCT coefficients of the observed signal $d_P(n)$ in the interval $(0 \le k \le P-L-1)$ are the same as those of the original signal $y_P(n)$.

Also, the number P is chosen as satisfying the inequality

$$P - M = z_\ell + z_r \geq L, \qquad z_\ell = z_r. \tag{1}$$

where z_ℓ and z_r are the number of zero values of $y_P(n)$ in the left and the right side, respectively. This inequality can be regarded as a convergence condition for the super-resolution method in the DCT domain and corresponds to one for the DFT-based scheme[6][7].

2.2.2 Simultaneous Linear Equations

The original signal $y_P(n)$ and the observed signal $d_P(n)$ are shown in matrix form as follows

$$\boldsymbol{y}_P = \boldsymbol{C}_P^{\mathrm{II}} \boldsymbol{Y}_P \tag{2}$$
$$\boldsymbol{d}_P = \boldsymbol{C}_P^{\mathrm{II}} \boldsymbol{D}_P, \tag{3}$$

where $\boldsymbol{y}_P$, $\boldsymbol{d}_P$, $\boldsymbol{Y}_P$ and $\boldsymbol{D}_P$ are the vectors of the original signal $y_P(n)$, the observed signal $d_P(n)$, their DCT coefficients $Y_P(k)$ and $D_P(k)$, respectively. $\boldsymbol{C}_P^{\mathrm{II}}$ is the P-point IDCT matrix of the type-II. From the assumption (**2**), the vectors of the DCT coefficients $\boldsymbol{Y}_P$ and $\boldsymbol{D}_P$ are the same except for $\boldsymbol{Y}_2$ and $\boldsymbol{D}_2$ as shown in Fig. 1. In the DCTs with other types, we only replace $\boldsymbol{C}_P^{\mathrm{II}}$ with the other IDCT matrix. As shown in Fig. 1, owing to the damaged DCT vector $\boldsymbol{D}_2$,

$$y_P(n) = 0, \qquad \begin{cases} 0 \le n \le z_\ell - 1 \\ z_\ell + M - 1 \le n \le P - 1 \end{cases} \tag{4}$$

cannot be guaranteed. To make the damaged vector $\boldsymbol{D}_2$ equal to $\boldsymbol{Y}_2$, the following equation has to be satisfied:

$$\begin{bmatrix} \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{C}_1 & \boldsymbol{C}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{Y}_1 \\ \hline \boldsymbol{Y}_2 \end{bmatrix}, \tag{5}$$

where $\boldsymbol{0}$ is the zero vector of order $P-M$. $\boldsymbol{C}_i (i = 1, 2)$ are sub-matrices of $\boldsymbol{C}_P^{\mathrm{II}}$ and the entry at row n and column k of these matrices is given by, respectively,

$$[\boldsymbol{C}_i]_{nk} = \begin{cases} \left[\boldsymbol{C}_P^{\mathrm{II}}\right]_{nk_i} & 0 \le n \le z_\ell - 1 \\ \left[\boldsymbol{C}_P^{\mathrm{II}}\right]_{(n+M)k_i} & z_\ell \le n \le P - M - 1 \end{cases} \tag{6}$$

$$k_i = \begin{cases} k, & 0 \le k \le P - L - 1 \quad (i = 1) \\ P - L + k, & 0 \le k \le L - 1 \quad (i = 2) \end{cases}$$

Now, we note that $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$ are known matrices and $\boldsymbol{Y}_1$ is also a known vector of the DCT coefficients. Thus, we can make simultaneous linear equations from Eq. (5) as

$$\boldsymbol{C}_2 \boldsymbol{Y}_2 = -(\boldsymbol{C}_1 \boldsymbol{Y}_1). \tag{7}$$

As a result, we see that reconstructing the original signal, that is estimating $\boldsymbol{Y}_2$ is reduced to solving Eq. (7). However, in general, the system of these linear equations is rank-deficient. That is, the number of linearly independent equations is less than the number of unknowns or the number of equations. The inverse matrix of $\boldsymbol{C}_2$ does not exist. Equation (7) is solved under the least-squares criteria as well as for the DFT-based scheme [5]. Thus, we can obtain the optimal solution $\widetilde{\boldsymbol{Y}}_2$ under this criteria, and the original signal $y_P(n)$ can be reconstructed by using $\widetilde{\boldsymbol{Y}}_2$.

3 MAGNIFICATION WITH THE DCT - BASED SCHEME

Next, we consider applying the proposed super-resolution method to the magnification of signals, namely signal interpolation. In the article[8], the super-resolution method with the DCT-II was applied to the magnification of images to improve image resolution. However, it was shown that the error between the original signal and the magnified signal diverges as the number of iteration increases[8][9]. We can derive two reasons for causing the divergence of the magnified signal, i.e., the one concerns with the applicable type of the DCTs and the other one is caused from the implementation procedure of the conventional scheme [9]. We will propose two magnification schemes to improve the conventional one[8].

In the following, from the discussion in Sect. 2.1, we describe the signal magnification under a type 1 LP FIR filter as a damage model.

3.1 The Proposed Magnification Schemes

We provide two magnification schemes based on the DCT-based scheme. We discuss the signal magnification for the up-sample factor $\mathcal{U}$=2 for convenience. We assume that the M points original signal $y(n)$ is band-limited, but is not necessarily band-limited to the region $|\omega|<\pi/2$ and the observed signal $d(n)$ is the down-sampled signal of $y(n)$ for the down-sample factor $\mathcal{D}$=2. In the following, we show two magnification schemes with the DCT-I or III[9](see Figs. 2, 3).

3.1.1 The Proposed Approach-1 (see Fig. 2)

Step 1: Up-sample the observed signal $d(n)$ by the up-sample factor $\mathcal{U}$ ($\mathcal{U}$=2) and obtain the M points signal $v(n)$.
Step 2: Do linear convolution of $v(n)$ with a filter $h_u(n)$ which is a lowpass filter band-limited to the region $|\omega|<\pi/\mathcal{U}$, and obtain the signal $w(n)$.
Step 3: Select the number P as satisfied Eq. (1) and add the zero values in the left and the right sides to $w(n)$ as shown in Fig. 2. Obtain the P points signal $w_P(n)$.
Step 4: Compute the P points DCT coefficients $W_P(k)$ of $w_P(n)$.
Step 5: Apply the proposed method in Sect. 2.2 to the signal $w_P(n)$ under the following two conditions: (**1**) It is known that the number of the original signal $y(n)$ is M. (**2**) The DCT coefficients corresponding to low frequency components of $w_P(n)$ are the same as those of the signal $y_P(n)$ which is P points signal extended from $y(n)$ as well as $w_P(n)$.

The signal magnification is completed by executing the above procedure. However, the accuracy of the obtained signal depends on the property of the filter $h_u(n)$ in **Step 2**. Thus, it is difficult to select the filter $h_u(n)$ to guarantee the assumption (**2**). Next, we propose another magnification scheme without any filters.

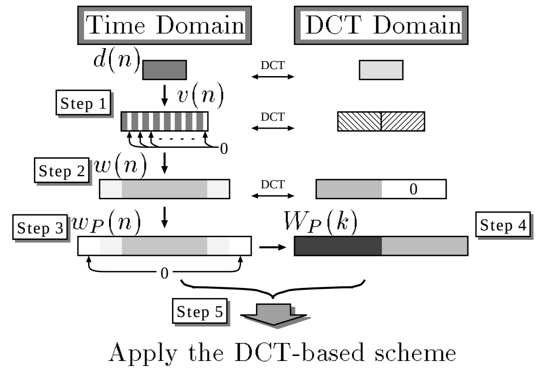

Figure 2: The implementation of *the proposed approach-1*

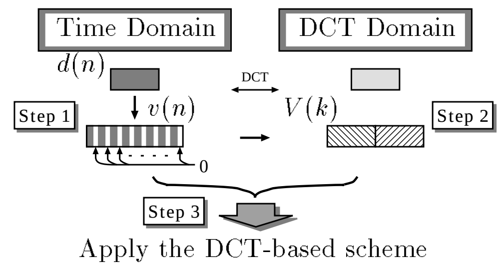

Figure 3: The implementation of *the proposed approach-2*

3.1.2 The Proposed Approach-2 (see Fig. 3)

Step 1: Execute the same as the *Approach-1*.
Step 2: Compute the M points DCT coefficients $V(k)$ of $v(n)$.
Step 3: Apply the proposed method in Sect. 2.2 to the signal $v(n)$ under the following three conditions: (**1**) The samples of $v(n)$ except for zero values, $d(n/\mathcal{U})$, are the same as those of the original signal $y(n)$. (**2**) The DCT coefficients corresponding to low frequency components of $v(n)$ are the same as those of $y(n)$. (**3**) The stopband region of the original signal $y(n)$ is known.

4 SIMULATION RESULTS

In order to verify the validity of the proposed super-resolution method, we provide the result of two computer simulations in Fig. 4 and Fig. 5, which show the convergence characteristics. As the reference of comparison we use the equation defined as

$$e_i = 10\, log_{10} \frac{||Y_P(k) - \widetilde{Y_i(k)}||}{||Y_P(k)||}, \tag{8}$$

where $Y_P(k)$ and $\widetilde{Y_i(k)}$ are the DCT coefficients of the original signal and its estimated signal after ith iteration and $||\cdot||$ denotes square norm.

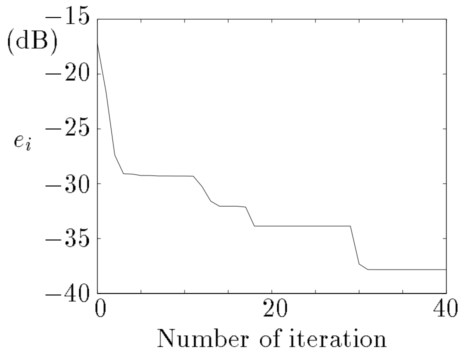

Figure 4: The convergence characteristics : Apply the DCT-based scheme for the observed signal damaged in the interval ($\pi/2<\omega<3\pi/2$).

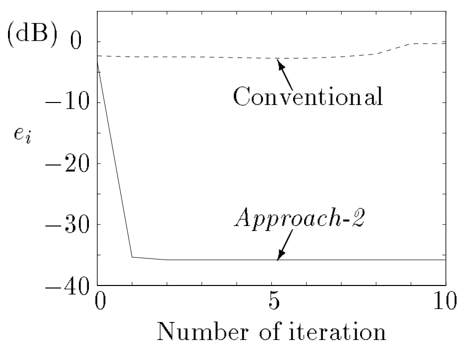

Figure 5: The convergence characteristics for the signal magnification : Comparison of *the proposed approach-2* and the conventional one.

First, we apply the DCT-based scheme provided in Sect. 2.2 to reconstructing an original signal from an observed signal with some frequency damage. The conditions of the simulations are as follows:

- The original signal $y_P(n)$ is a signal added zero values in the left and the right sides to an M=16 points signal taken from the 110th row of the "Barbara' image. The number of the added zero values and $y_P(n)$ is z_ℓ=z_r=8 and P=32, respectively.
- The observed signal $d_P(n)$ is damaged in the interval ($\pi/2<\omega<3\pi/2$). The number of the damaged DCT coefficients is L=16.
- We apply the DCT-based scheme with the DCT-II, which solves Eq. (7) by the conjugate gradient method, for $d_P(n)$.

In Fig. 4, the convergence characteristics of the proposed method is shown. From this result, we can see that the original signal $y(n)$ is reconstructed.

Next, we verify the validity of applying the *Proposed Approach-2* with DCT-I to the signal magnification. Figure 5 shows the result of applying the *Approach-2* and that of applying the conventional procedure[8] under the conditions:

- The original signal $y(n)$ is taken from the "Barbara" image and band-limited by a lowpass filter with cutoff frequency $2\pi/3$. The number of $y(n)$ is 17.
- The observed signal $d(n)$ is the down-sampled signal of $y(n)$ for the down-sample factor $\mathcal{D}$=2. The number of $d(n)$ is 9.
- *The Approach-2* with the 17-point DCT-I and the conventional procedure are executed for the up-sample factor $\mathcal{U}$=2, respectively.

From Fig. 5, we can verify that the *Proposed Approach-2* provides the convergence of the error between the original signal and the magnified signal, while the conventional procedure not. As well as the DCT-I, we can derive the similar result for the *Proposed Approach-2* with the DCT-III.

5 CONCLUSION

In this paper, we proposed a new super-resolution method based on the DCT for a signal with some frequency damage. The proposed method can be carried out with real value operation and is applicable to any DCT in 4 kinds of DCTs under a type 1 LP FIR filter as a damage model. Besides, we expanded the proposed super-resolution method to two magnification schemes with the DCT-I or III. Finally, we showed the result of computer simulations. From the results, we verified that the proposed super-resolution method achieves to reconstruct original signals.

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