DETECTION OF ABRUPT CHANGES : A TIME-FREQUENCY APPROACH

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ABSTRACT

This paper presents a comparison between parametric and non-parametric approaches of abrupt changes detection in noisy signals. The goal is to propose an alternative way to be used when the model-based methods do not work very well because of an unsuitable model structure or a non strictly stationnary stepwise signal. In this latter case, an analysis of time-frequency distributions allows the detection of abrupt spectral changes without any hypothesis and provides some results as good as parametric methods for the studied type of signals.

1 INTRODUCTION

Detection of abrupt spectral changes in noisy signals is an important and well studied problem. The field of applications covers watching for industrial plants as well as medical monitoring or collision warning... The standard methods use a model of the signal (harmonic, AR, ARMA) and can roughly be classified in two sets : in the first one, a prediction error is built up from the signal by a suitable filter and an abrupt spectral change in the signal involves a simple level change in a statistic deduced from the prediction error. The second set gathers the methods using comparisons of signal models, identified on two (or three) observation windows of different lengths. If a change occurs, a model issued from large-windowed data temporarily remains constant, while a short-windowed one quickly changes. The detection process consists then in comparing the two previous models and deciding if they are different. Unfortunately all model-based methods need two conditions : firstly, the model structure has to be well adapted to the signal, that is sometimes difficult in practical cases (biomedical signals for example). Secondly, the time delay between two changes must be sufficiently long to allow a good parameter estimation, even if the random nature of the estimates is taken into account. For these reasons, a non parametric approach based on time-frequency representations (TFRs) is proposed. The basic idea of this method is the following: As well as for parametric approaches, where a long-time and short-time model are compared, two smoothed time-frequency distributions are compared again. Roughly speaking, the two temporal windows (large and short) involve same low level energy cross-terms if no change occurs. Front of an abrupt change, those cross-terms become very different and the instantaneous distance between the two TFRs sharply peaks at the change time. In the paper, the last results concerning TFRs distance measures are presented. Then the non parametric proposed method is given. Some simulation tests are exhibited and the results are compared with classical parametric approaches.

2 TEST SIGNALS

Very simple signals have been considered first to illustrate our study. They are presented in Figure 1. All the signals are 128 data length (long=128). $x_1(t)$ (at the top of the figure) consists in two sinusoids s_1 and s_2, s_1 with a normalized frequency f1 on the time-domain [0,t1[∪[t2,long] and s_2 with a normalized frequency f2 on the time-domain [t1,t2[. $x_2(t)$ (at the bottom) is a sinusoid with a normalized frequency f1. Here f1=0.2, f2=0.15, t1=50, t2=90 and the signal-to-noise ratio has been fixed to 5db.

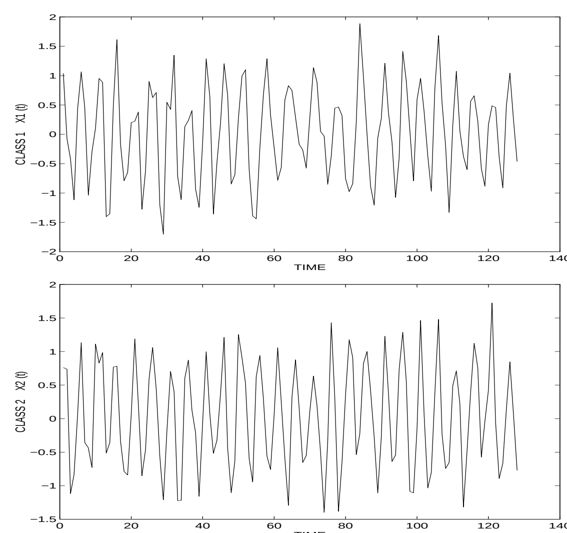


Figure 1: **Test signals**

3 NON-PARAMETRIC APPROACH

The choice of the couple (TFR/distance measure between TFRs) is very important [1], [2]. From a recent work performing a comparison between different associations (TFR/distance), the choice ($|SPWD|$,Kolmogorov distance), where SPWD is the smoothed pseudo-Wigner distribution, seems to give good results for the considered signals and will be used in this paper. All the presented TFRs are computed from analytic signals and have been normalized. A broader time smoothing function will produce more time smoothing and thus better cross-terms (CT) softening but poorer time resolution. If we consider two TFRs of signal $x_1(t)$ using two time smoothing windows (lengths $Lt_1 = 11$, $Lt_2 = 65$) and the same frequency smoothing window (Lf=125), we observe the same low energy cross-terms during the intervals where no change occurs. In front of abrupt changes, the cross-terms become very different. This property is shown on Figure 2. The idea is thus to use the CT to underscore the presence of abrupt changes. By comparing the two TFRs, we define the distance (Kolmogorov like) :

$$\mathrm{D_K}(\mathrm{TRF_1}, \mathrm{TFR_2}) = \int\int \mid |\mathrm{TFR_1}| - |\mathrm{TFR_2}| \mid \mathrm{dtdf} \quad (1)$$

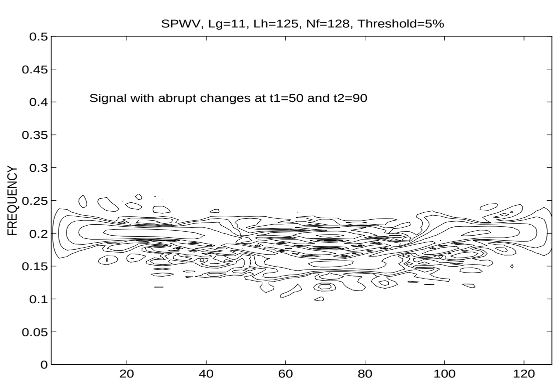
and the instantaneous distance $\Delta(t)$ is defined by:

$$\Delta(t) = \int \mid |TFR_1| - |TFR_2| \mid df \quad (2)$$

so that :

$$D_K(TRF_1, TFR_2) = \int \Delta(t)\, dt \quad (3)$$

Figure 3 presents the evolution of $\Delta(t)$. For $x_2(t)$ (without changes), $\Delta(t)$ is nearly constant. For $x_1(t)$, $\Delta(t)$ sharply peaks at the change times.



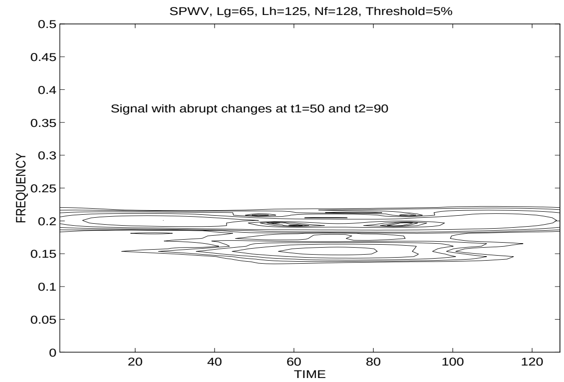


Figure 2: **Signal with abrupt changes** $SPWD_2$(Lt=11,Lf=125), $SPWD_1$(Lt=65,Lf=125)

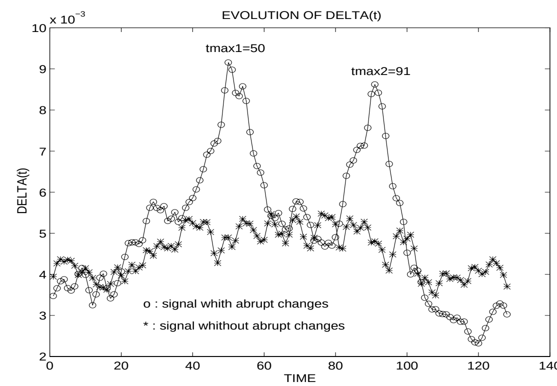


Figure 3: **Evolution of** $\Delta(t)$

4 PARAMETRIC APPROACH

We will also propose a comparison with parametric approaches, based on the use of two classical detection procedures : the first one tests the whiteness property of the signal innovation provided by a Kalman filter [5] [6] and the second one considers the distance measure between two models identified within two different length observation windows [7]. We briefly recall both of the methods.

4.1 Whiteness

The processed signal $x[t]$ is assumed to be a sinusoid :

$$x[t] = sin(\phi[t]) + v[t] \quad (4)$$

where $v[t]$ is a centered gaussian sequence of unknown variance σ_v^2 and $\phi[t]$ is the linear phase.
Let's now define the state vector :

$$\theta[t]^T = (\phi[t], 2\pi\lambda[t])^T \quad (5)$$

where the frequency $\lambda[t]$ is :

$$\lambda[t] = (\phi[t+1] - \phi[t])/(2\pi) \quad (6)$$

These hypothesis lead straigth on the recursive linear equation :

$$\theta[t+1] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \theta[t] \quad (7)$$

If the Tretter approximation is verified (i.e. SNR>5dB), the additive noise $v[t]$ is converted into a modulation phase noise $b[t]$ and we associate to the state-space model the following measure equation :

$$arg(z[t]) = H\theta[t] + b[t] \quad (8)$$

where $H = [1\ 0\ 0]$ and z[t] is the analytic signal deduced from x[t].
We are able now to apply a linear Kalman filter and the signal innovation $\epsilon[t]$ is then defined by :

$$\epsilon[t] = arg(z[t]) - H\hat{\theta}_{t/t} \quad (9)$$

$\hat{\theta}_{t/t}$ denotes the current identified model.
The whiteness of the signal innovation is finally tested by computing an ergodic estimation of the first point of its correlation function. This estimation is given by the following recursive equation :

$$\rho_k = \alpha\rho_{k-1} + (1-\alpha)\epsilon_k\epsilon_{k-1} \quad (10)$$

where α is a constant factor $(0 \leq \alpha \leq 1)$.
If the absolute estimated value becomes greater than a threshold (*a priori* fixed), then the presence of a change is decided. The first change is detected very quickly.

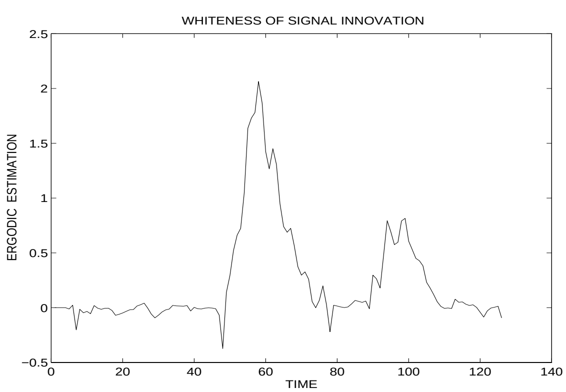


Figure 4: **Whiteness of the signal innovation**

Considering that the algorithm is going on without computing any other identification before the second abrupt changes, the results are quite good.

4.2 Distance between two uncertain models

The second detector uses a distance measure between two models identified within different length observation windows : a reference large one L_0 which takes into account all (or almost all) the past of the signal, and a test short one which only contains the L_1 past observations ($L_1 \approx 20$). As a result, if no change occurs,

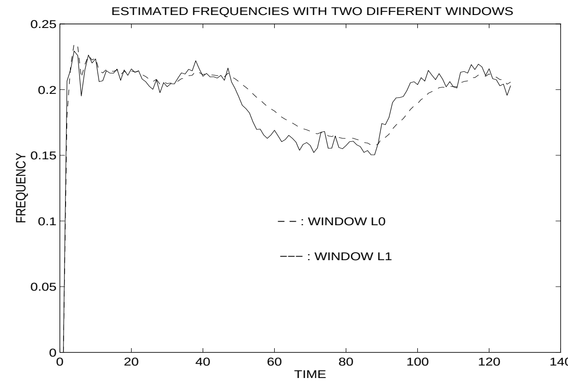


Figure 5: **Estimated frequencies**

the long-term windowed observations lead to the same model than the short term ones. Noting $\hat{\theta}_{0t/t}$ and $\hat{\theta}_{1t/t}$ the respective estimated parameter vectors, $P_{0t/t}$ and $P_{1t/t}$ the estimation error variance-covariance matrices, the distance measure is defined by :

$$\begin{aligned} d[t] &= (\hat{\theta}_{1t/t} - \hat{\theta}_{0t/t})^T ... \\ &...((1+\frac{L_1}{L_0})P_{1t/t})^{-1}(\hat{\theta}_{1t/t} - \hat{\theta}_{0t/t}) \quad (11) \end{aligned}$$

If $d[t]$ becomes greater than a threshold (*a priori* fixed), then the presence of a change is decided. Depending

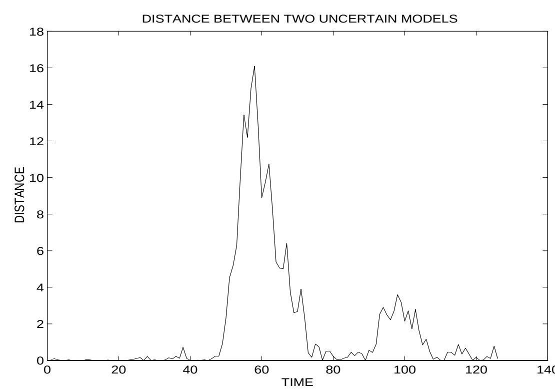


Figure 6: **Distance between two uncertain models**

on the length observation windows, the frequencies are more or less well estimated when a change occurs. However inn the case of a short window, we note that the frequency already gives an idea of the abrupt changes. If we consider the distance between the two different windows, it is quite easy to localize the first abrupt changes. The second one is more difficult to detect. To be correct, we should compute another identification just after the abrupt changes in order to be in the same conditions of detection as for the first one.

5 CONCLUSION

The choice between a parametric or a non parametric approach is conditioned by some constraints. We pro-

pose in conclusion some problems which confront the user for deciding which method he should compute.
Concerning the parametric approach, the difficulty in a real case is in defining the model. If the model is well adapted and if the identification is achieved, you perform the abrupt changes detection algorithm in very good conditions. Excepted that it should be necessary to identify the model between every abrupt changes.
Concerning the non-parametric approach, many questions, of course, must be answered : choice of the time-frequency distribution and of the smoothed versions in connection with the cross-terms geometry (Cohen, Hyperbolic, k-power, others...); choice of the distance measure between distributions (Rényi, Kolmogorov...) [3], [4]; class of suitable signals...
Depending on the possibility to modelize the process with high precision and the stationnarity of the signal between the abrupt changes, it could very be usefull to start with a non-parametric approach to get an idea of the model and the nature of non-stationnarity. Then if possible you try a parametric approach.
It must be underlined that this work is one of the first dealing with change detection in the time-frequency plane. As a consequence, many questions remain opened and many expansions are possible.

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