MMSE EQUALIZERS FOR MULTITONE SYSTEMS WITHOUT GUARD TIME

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ABSTRACT

Recently the concept of multitone modulation or OFDM has received much attention. For such a modulation, the dispersiveness of the channel is classically solved by the technique of guard time. In the present paper we investigate the performance of OFDM without guard time but with MIMO equalization. Linear and decision-feedback structures structures are derived for an MMSE criterion and their performance is assessed by means of their steady-state behavior. Symbol rate equalizers following channel matched filters are derived and investigated. It is shown that equalized OFDM outperforms OFDM with guard time.

1. INTRODUCTION

Recently the technique of multitone modulation or OFDM (Orthogonal Frequency Division Multiplexing) has gained a lot of interest [2]. The basic idea of OFDM is to serial-to-parallel (S/P) convert the input symbol stream and then modulate several carriers in parallel. The carrier frequencies are selected so as to be orthogonal on the symbol duration. After S/P conversion each individual symbol stream requires a narrower bandwidth around its associated carrier. Hence the bandwidth necessary for the OFDM modulated signal is about the same as that that would be required by single carrier modulation with the input symbol stream. If the number of carriers goes up, assuming frequency synchronization is still possible, the spectrum occupied by the OFDM signal would tend to a perfect rectangle. When the channel is not dispersive, although the spectra of the different carriers overlap, each symbol stream can be recovered without inter-carrier interference (ICI). When the channel is dispersive, without any particular care, the received OFDM signal will be affected by ISI (inter symbol interference) and also ICI, because the orthogonality between tones is lost in the most general case. A very elegant way to mitigate the delay spread associated with the channel impulse response is to use a guard time [1]. After demodulation each symbol needs only be corrected by a complex coefficient associated with the channel transfer function. A first penalty associated with that technique is the fact that the integration performed by the matched filter only uses a fraction of the received energy. However, this penalty becomes very negligible when large numbers of tones are used. A second penalty which is more critical is that the different subchannels become frequency nonselective and hence it is not possible to perform an efficient detection of those symbols that are transmitted on a deeply faded carrier [4]. Frequency interleaving together with error correction coding is mandatory to avoid this effect. Another possibility is to design a receiver able to handle the signal corrupted by the dispersiveness of the channel. In the present paper it will be shown that the complete system between the sequences of symbols up to the sampling of matched filter outputs can be modeled by a MIMO (multiple input-multiple output) digital equivalent system. Therefore, suboptimal receivers can be based on MIMO equalizers connected to the matched filters. Linear and decision-feedback (DF) devices will be considered. The derivations will be made for multitone modulation in the direct domain and the coefficients will be explicitly given. For the two types of structures, it will be shown how to compute the exact BER (bit error rate) corresponding to the steady-state. The performance of the equalizers will be investigated for a typical two-ray channel. They will be compared to the performance achieved by means of the technique of guard time.

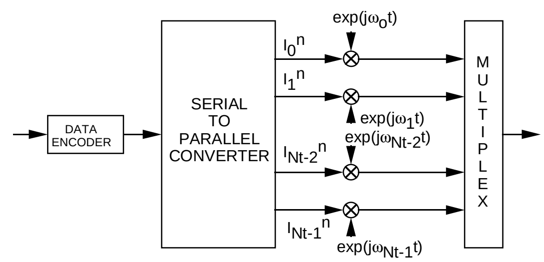

Figure 1. Transmitter of a multitone system

2. MULTITONE SYSTEMS

2.1. Basic system

The principle of a multitone transmitter is depicted in figure 1.

The input symbol stream is S/P converted and the different symbol streams modulate carriers which are orthogonal on the symbol duration. Hence, assuming N_t carriers, the lowpass equivalent transmitted signal is given by

$$x(t) = \sqrt{\frac{2P}{N_t}} \sum_{p=0}^{N_t-1} \sum_{n=-\infty}^{\infty} I_p^n \, u(t-nT) \, \exp^{2\pi jpt/T} \tag{1}$$

where I_p^n is the nth symbol conveyed by carrier p. In the present paper we assume to have BPSK modulation. The symbol shape $u(t)$ is assumed to be rectangular. The RF frequency associated with the pth carrier is given by f_0+p/T where f_0 is some base frequency and T is the symbol duration. The factor $1/\sqrt{N_t}$ is introduced to keep the overall transmission power constant whatever the number of tones.

Assuming a linear channel with equivalent lowpass impulse response $c(t)$, the lowpass received signal is given by

$$r(t) = \sqrt{\frac{2P}{N_t}} \sum_{p=0}^{N_t-1} \sum_{n=-\infty}^{\infty} I_p^n \, h_p(t-nT) + n(t) \qquad (2)$$

where $u_p(t) = u(t) \exp^{2\pi jpt/T}$, $h_p(t) = u_p(t) \otimes c(t)$, and $\otimes$ denotes convolution, $n(t)$ is the AWGN with one-sided power spectral density N_0. The matched filter output (assuming perfect carrier phase and sampling time recovery) is defined by

$$y_p^n = \frac{1}{T\sqrt{2P}} \int_{-\infty}^{\infty} r(t) \, h_p^*(t-nT) \, \mathrm{d}t \qquad (3)$$

with

$$x_{p,p'}^{n-n'} = \frac{1}{T} \int_{-\infty}^{\infty} h_p^*(t-nT) \, h_{p'}(t-n'T) \, \mathrm{d}t \qquad (4)$$

It turns out that

$$y_p^n = \sum_{q=0}^{N_t-1} \sum_{m=-\infty}^{\infty} I_q^m \, \frac{x_{p,q}^{n-m}}{N_t} + \frac{\nu_p^n}{T\sqrt{2P}} \qquad (5)$$

where ν_p^n represents samples of the filtered white noise. From equation 5, it appears that the overall system between the symbol generation up to the sampling of the matched filter outputs can be modeled by a MIMO digital equivalent system. It also appears that the samples at the outputs of the matched filters are affected not only by ISI (inter symbol interference) but also by inter carrier interference (ICI). This suggests that the equalization structure also has to be of the MIMO type. Furthermore, the possible complex valued interference terms would imply I-Q interference in the case more general QAM constellations would be used.

2.2. Technique of guard time

If the channel impulse response has a length limited to T_g, a guard time of T_g seconds can be used. Assuming that the lowpass equivalent impulse response $c(t)$ is non-zero for $t \in [0, T_g]$ only, it can be shown that

$$y_{g,q}^n = \sqrt{\frac{2P}{N_t}} \sum_{p=0}^{N_t-1} I_p^n \, C(2\pi p/T) \, T\delta(p,q) + \nu_{g,q}^n \qquad (6)$$

where $C(\omega)$ is the transfer function of $c(t)$ and the noise term $\nu_{g,q}^n$ has a variance $2N_0T$. It turns out that at the output of the matched filter bank the symbol are free from ICI but not from I-Q interference. On the other hand, there is a penalty incurred by this technique due to the fact that the integration interval in the matched filter is T rather than $T+T_g$. This is required to benefit from the orthogonality properties. Assuming that the channel transfer function can be evaluated, zero forcing equalization (meaning cancellation of the I-Q interference) is achieved by dividing the $y_{g,q}^n$ by the corresponding $C(2\pi q/T)$ factor. Hence symbol estimates are given by

$$\hat{I}_{g,p}^n = I_p^n + \frac{\nu_{g,q}^n}{C(2\pi q/T)T\sqrt{2P/N_t}} \qquad (7)$$

The symbol estimates are Gaussian if the channel is known. Assuming BPSK symbol transmission, the bit error probability for carrier q is given

$$P_{e,q} = 0.5 + 0.5 * \operatorname{erf}\left[C(2\pi q/T)C^*(2\pi q/T)E_b/N_0\right]^{-0.5} \qquad (8)$$

with $E_b = PT/N_t$.

3. LINEAR MMSE EQUALIZATION

3.1. Steady-state solution

The goal of an MMSE (Minimum Mean Square Error) MIMO equalizer is to find coefficients $c_{r,s}^m$ such that the expectation of the error between the true symbols I_p^n and their prediction $\hat{I}_p^n$ built from the sampled matched filter outputs is minimum. If we assume $2K+1$ coefficients in each branch of the equalizer, the prediction is computed as

$$\hat{I}_q^m = \sum_{p=0}^{N_t-1} \sum_{n=-K}^{K} c_{q,p}^n \, y_p^{m-n} \qquad (9)$$

The coefficients have to be such that they minimize

$$\frac{1}{N_t} \sum_{p=0}^{N_t-1} E\left[|\epsilon_p^k|^2\right] = \frac{1}{N_t} \sum_{p=0}^{N_t-1} E\left[|I_p^k - \hat{I}_p^k|^2\right] \qquad (10)$$

According to the results reported in [3], we use the orthogonality principle and require that

$$E[\epsilon_q^k (y_p^{k-l})^*] = 0 \qquad (11)$$

for all combinations of p and q and all values of l. If we limit the length of the different filters to $2K+1$ coefficients, these filter coefficients have to fulfill a set of $N_t \times N_t \times (2K+1)$ equations :

$$\sum_{r=0}^{N_t-1} \sum_{m=-K}^{K} c_{q,r}^m \quad \times$$
$$\left[\sum_{s=0}^{N_t-1} \sum_{n=-\infty}^{\infty} \frac{x_{r,s}^n (x_{p,s}^{n+m-l})^*}{N_t} + \frac{x_{r,p}^{l-m}}{N_t(E_b/N_0)}\right] = \frac{(x_{p,q}^{-l})^*}{\sqrt{N_t}} \qquad (12)$$

where N_0 is the one-sided power spectral density of the AWGN, E_b/N_0 is the energy per bit over white noise ratio and $E_b = PT/N_t$. This equation can be written in a matrix form and it is worthwhile mentioning that the matrix coefficients do not depend on q. This means that this matrix can be inverted once.

3.2. Bit error rate

The residual interference after equalization can be computed by

$$\hat{I}_q^n = \sum_{p=0}^{N_t-1} \sum_{m=-\infty}^{\infty} I_p^m \, r_{q,p}^{n-m} + \sum_{p=0}^{N_t-1} \sum_{m=-K}^{K} c_{q,p}^m \, \frac{\nu_p^{n-m}}{T\sqrt{2P}} \qquad (13)$$

where

$$r_{q,p}^n = \sum_{t=0}^{N_t-1} \sum_{m=-K}^{K} c_{q,t}^m \, \frac{x_{t,p}^{n-m}}{\sqrt{N_t}} \qquad (14)$$

The bit error probability can be computed by means of an approach based on the characteristic function [5]. The bit error probability at the output of the equalizer can be computed from this residual interference. As the equalization coefficients involved in the equalization process may be complex, the decision variable can also be complex, even if the transmitted symbols are real. As a consequence, the bit error probability (after equalization) is defined as the probability that the real part of the decision variable (after equalization) is negative assuming that a "1" has been transmitted :

$$\mathrm{P}_e = \mathrm{P}\left[\Re\left(\hat{I}_q^n\right) \leq 0 | I_q^n = 1\right] \tag{15}$$

It can be shown that this BER can be computed as

$$\begin{aligned}\mathrm{P}_{e,q} &= \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \mathrm{d}\omega_1 \frac{\sin\left[\omega_1 \Re(r_{q,q}^0)\right]}{\omega_1}\, e^{-\omega_1^2 \sigma_{N,q}^2/4} \\ &\times \prod_{p=0}^{N_t-1} \prod_{m=-\infty}^{\infty} \cos\left[\omega_1 \Re(r_{q,p}^{n-m})\right]\end{aligned} \tag{16}$$

In the double product, when $p = q$, the value of $m = n$ does not have to be taken into account because the value of I_q^n is assumed and

$$\sigma_{N,q}^2 = \sum_{p=0}^{N_t-1} \sum_{p'=0}^{N_t-1} \sum_{m=-K}^{K} \sum_{m'=-K}^{K} c_{q,p}^m \, (c_{q,p'}^{m'})^* \, \frac{x_{p,p'}^{m'-m}}{N_t(E_b/N_0)} \tag{17}$$

4. MMSE DFE EQUALIZATION

4.1. Steady-state solution

The goal of an MMSE (Minimum Mean Square Error) MIMO DF equalizer is to find coefficients $c_{r,s}^m$ such that the expectation of the error between the true symbols I_p^n and their prediction $\hat{I}_p^n$ built from the sampled matched filter outputs and the decisions about previous symbols is minimum. In this work, we assume that the decisions taken about the previous symbols are correct and hence we identify the decisions with the true symbols. If we assume K_1 and K_2 coefficients in the forward and the feedback sections respectively, the prediction computed by the DF equalizer is

$$\hat{I}_q^m = \sum_{p=0}^{N_t-1} \sum_{n=-K_1}^{0} c_{q,p}^n \, y_p^{m-n} + \sum_{p=0}^{N_t-1} \sum_{n=1}^{K_2} c_{q,p}^n \, I_p^{m-n} \tag{18}$$

The solution of the MMSE design is a bit more involved than for linear equalization [6]. The partial derivatives with respect to the coefficients depend on whether the coefficient belongs to a forward or a feedback section. For the forward section, we find that the coefficients have to fulfill

$$E[\epsilon_q^k (y_p^{k-l})^*] = 0 \qquad \text{for } -K_1 \leq l \leq 0 \tag{19}$$

For the feedback section, one obtains

$$E[\epsilon_q^k (I_p^{k-l})^*] = 0 \qquad \text{for } 1 \leq l \leq K_2 \tag{20}$$

for $q \in [0, N_t - 1]$, $p \in [0, N_t - 1]$ and all possible values of l compatible with the section under consideration. Considering that the transmitted symbols are all independent, equation 19 leads to a set of $(K_1 + 1) \times N_t^2$ equations. For $q \in [0, N_t - 1]$, $p \in [0, N_t - 1]$ and $-K_1 \leq l \leq 0$ the coefficients of the forward section have to fulfill

$$\begin{aligned}&\sum_{r=0}^{N_t-1} \sum_{m=-K_1}^{0} c_{q,r}^m \quad \times \\ &\left[\sum_{s=0}^{N_t-1} \sum_{n=-\infty}^{\infty} \frac{x_{r,s}^n (x_{p,s}^{n+m-l})^*}{N_t} + \frac{x_{r,p}^{l-m}}{N_t(E_b/N_0)}\right] \quad + \\ &\sum_{r=0}^{N_t-1} \sum_{m=1}^{K_2} c_{q,r}^m \frac{(x_{p,r}^{m-l})^*}{\sqrt{N_t}} \quad = \quad \frac{(x_{p,q}^{-l})^*}{\sqrt{N_t}}\end{aligned} \tag{21}$$

For $q \in [0, N_t - 1]$, $p \in [0, N_t - 1]$ and $1 \leq l \leq K_2$ the coefficients of the feedback section have to fulfill

$$\sum_{r=0}^{N_t-1} \sum_{m=-K_1}^{0} c_{q,r}^m \frac{x_{r,p}^{l-m}}{\sqrt{N_t}} = -c_{q,p}^l \tag{22}$$

where N_0 is the one-sided power spectral density of the AWGN and $E_b/N_0 = PT/N_t$ is the symbol energy over white noise power spectral density ratio. The same remarks as those given for the linear equalizers hold concerning the matrices.

4.2. Bit error rate

The residual interference can be computed as previously with

$$\begin{aligned} r_{q,p}^n &= 0 \text{ for } p \in [0, N_t - 1] \text{ and } 1 \leq n \leq K_2 \\ r_{q,p}^n &= \sum_{t=0}^{N_t-1} \sum_{m=-K_1}^{0} c_{q,t}^m \, x_{t,p}^{n-m}/\sqrt{N_t} \text{ elsewhere}\end{aligned} \tag{23}$$

It is an extension of the classical behavior of DFE equalizers : *if the feedback section spans K_2 symbols, the interference from the K_2 previously detected symbols of neighboring tones is perfectly removed.* The characteristic function approach can still be used. The variance of the additive noise is computed by

$$\sigma_{N,q}^2 = \sum_{p=0}^{N_t-1} \sum_{p'=0}^{N_t-1} \sum_{m=-K_1}^{0} \sum_{m'=-K_1}^{0} c_{q,p}^m \left(c_{q,p'}^{m'}\right)^* \frac{x_{p,p'}^{m'-m}}{N_t(E_b/N_0)} \tag{24}$$

5. MULTIPATH CHANNEL

5.1. Channel description

The purpose of this section is to apply the results obtained in the preceding sections to a multipath channel. In order to understand the effects of the different parameters, we already mentioned that we want to investigate the case of a 2-paths channel, so that the set of parameter is limited to the gain and delay of the second path. We assume an impulse response given by

$$c(t) = \delta(t) + \beta\delta(t - \tau) \tag{25}$$

If the delay is written as $\tau = sT + \nu$, with s integer and $0 \leq \nu \leq T$, we obtain after normalization that

$$\begin{aligned} x_{p,q}^n &= \delta(n)\, \delta(p - q)\, (1 + \beta\beta^*) \\ &+ \beta^* \exp^{2\pi j p\nu/T}\end{aligned}$$

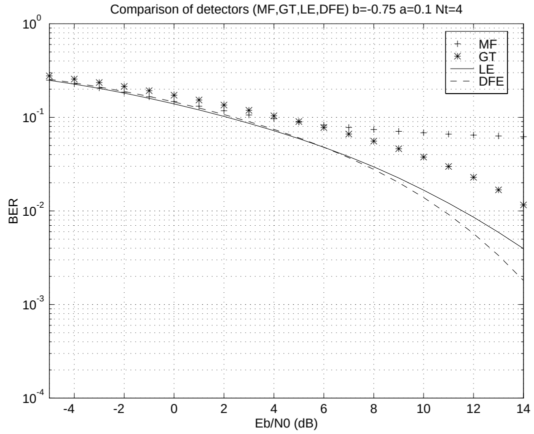

Figure 2. Comparison of multitone transmission with matched filter detection, guard time, linear and DF equalization. The parameters are $\beta = -0.5$, $\tau = 0.1T$, $N_t = 4$. The average among the different tones are shown. The $+$ are the results for matched filter detection, the $*$ for guard time, the $-$ for linear equalization and the $--$ for DF equalization.

$$
\begin{aligned}
\times & \left[\delta(n+s)\frac{\hat{R}_{p,q}(\nu)}{T} + \delta(n+s+1)\frac{R_{p,q}(\nu)}{T}\right] \\
+ & \beta \exp^{-2\pi j q \nu / T} \\
\times & \left[\delta(n-s-1)\frac{R_{p,q}(\nu)}{T} + \delta(n-s)\frac{\hat{R}_{p,q}(\nu)}{T}\right] \qquad (26)
\end{aligned}
$$

The correlations are defined as follows :

$$
R_{p,q}(\nu) = \int_0^{\nu} \exp^{-2\pi j (p-q) t/T} \, \mathrm{d}t \qquad (27)
$$

$$
\hat{R}_{p,q}(\nu) = \int_{\nu}^{T} \exp^{-2\pi j (p-q) t/T} \, \mathrm{d}t \qquad (28)
$$

We are of course aware that to be able to compute these $x^n_{p,q}$, the values of β and τ need to be estimated and their estimation will affect the system performance. However, we are interested in knowing the potential of the system and its performance in ideal conditions.

5.2. Computational results

As an illustrative example, a channel has been considered where $\beta = -0.75$ and $\tau = 0.1T$. Such a small value of the delay of the second path has to be selected in order to make the technique of guard time relevant. We have compared the efficiency of four different detectors : the decisions taken from the matched filter outputs, the equalization by guard time, and then the linear and DF equalizers. Figure 2 shows the results for a system with $N_t = 4$ tones. The $+$ are for the matched filter detection, the $*$ for the guard time technique, the $-$ for the linear equalizer, and the $--$ for the DF equalizer. The results clearly show that not using a guard time but linear or DF equalization provides better performance. There is a gain of about 2.5dB at a BER of 10^{-2} provided by the MIMO equalizers. Besides the figure also illustrates the additional gain provided by having decision-feedback. The linear and DF equalizers derived in the present paper do not assume that a guard time is used, nor, of course, that the delay (or the length of the channel impulse response) is short compared to the symbol duration. This is not true for the guard time equalizer which is no longer relevant when the channel impulse response is not short compared to the symbol duration. Hence we also have investigated the performance of linear and DF equalizers for $N_t = 4$, $\tau = 1.75T$, $\beta = -0.5$. The performance is shown by figure 3. It clearly appears that such equalizers are effective, even when the technique of guard time is not relevant. Besides the figure also illustrates the additional gain brought by DF equalization.

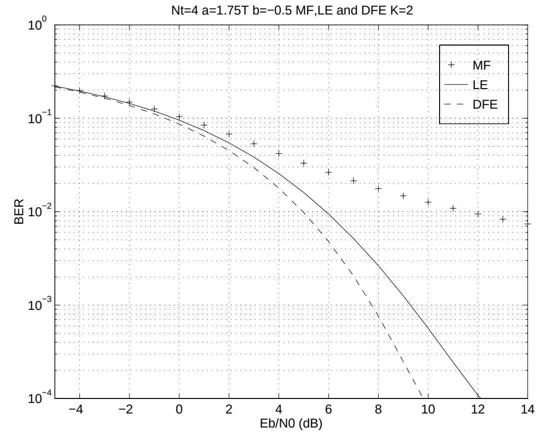

Figure 3. Comparison of multitone transmission with matched filter detection, linear and DF equalization. The parameters are $\beta = -0.5$, $\tau = 1.75T$, $N_t = 4$. The average among the different tones are shown. The $+$ are the results for matched filter detection, the $-$ for linear equalization and the $--$ for DF equalization.

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