CHANNEL EQUALIZATION USING PARTIAL LIKELIHOOD ESTIMATION AND RECURRENT CANONICAL PIECEWISE LINEAR NETWORK

Xiao Liu and Tülay Adalı
Information Technology Laboratory
Department of Computer Science and Electrical Engineering
University of Maryland Baltimore County
Baltimore, MD 21228-5938, USA Tel: (410) 455-3521; fax: (410) 455-3969
e-mail: xliu@engr.umbc.edu adali@engr.umbc.edu

ABSTRACT

A recurrent canonical piecewise linear (RCPL) network is proposed based on the canonical piecewise linear (CPL) structure and is applied to channel equalization. RCPL network provides savings in computation and implementation and has a distinct dynamic behavior completely different than that of finite duration feedforward structure. The simulations of multilevel signal equalization demonstrate the superior performance of RCPL equalizer when compared to the multilayer perceptron equalizer. For the RCPL network, it is easy to incorporate the a-priori information into the network structure. A novel blind algorithm is presented by combining partial likelihood estimation and RCPL structure for the binary communications channel. The simulation results show that RCPL blind equalizer outperforms the CMA equalizer by orders of magnitude for blind equalization of nonlinear communication channels .

1 INTRODUCTION

Recently, a number of neural network equalizers have been introduced to the demands of today's communication applications, particularly to compensate for nonlinear time varying distortion. It is shown that neural network equalizers can successfully equalize nonlinear channels where linear equalizers might fail. However, the most popular multilayer perceptron (MLP) equalizer [6] requires a large amount of training time and large network size. Recurrent neural network equalizer [8] can accurately model the inverse of a nonlinear communication channel with smaller network size, but for blind equalization, it is very difficult to incorporate statistics information into the structure because of the highly nonlinear structure of RNN. In this paper, we propose recurrent canonical piecewise-linear neural network equalizer which is based on the *canonical piecewise-linear structure* [9]. Hence, while incorporating recurrence in its structure, it can also overcome the problems associated with RNN equalizer because of its piecewise linear structure. Specifically, The RCPL network offers following benefits: (1) It provides savings in computation and implementation, especially when required to model strong nonlinearities. (2) The resulting algorithm is always stable. (3) Because of its piecewise linearities, it is easy to incorporate the a-priori statistical information into the equalizer structure. (4) Since RCPL network also employs feedback, it has a distinct dynamic behavior which is completely different from that attained by the use of finite duration impulse response feedforward structures. This is demonstrated by the superior performance of RCPL network for multilevel signal equalization in our simulation studies.

Blind equalizers are a special class of equalizers that determine their parameters based on the statistics of the channel input and the measured output when training sequences are not accessible. Since many different blind equalizers such as the Bussgang techniques [7], are developed for linear channels, the use of these algorithms will suffer from a severe performance degradation for unknown nonlinear channels. In this paper, we introduce a novel algorithm for blind equalization of nonlinear binary communications channel by combining partial likelihood estimation [1] with RCPL network. We have recently presented a general formulation for adaptive equalization by distribution learning [1], [2] in which conditional probability mass function (pmf) of the transmitted signal given the received signal is parameterized by a general neural network structure. The parameters of the pmf are estimated by minimization of the accumulated relative entropy (ARE) cost function. The equivalence of ARE minimization to maximum partial likelihood (MPL) estimation can be established under certain regularity conditions [1]. This equivalence enables us to use unsupervised learning to bypass the requirement that the true conditionals be known and to obtain large sample properties of the estimator for a general conditional pmf model without making the assumption of independent observations. On the other hand, RCPL neural network has the ability to learn nonlinear mappings of arbitrary complexity and to incorporate the known statistical information into the structure. Thus, the combination of partial likelihood estimation with RCPL neural network provides us with a unique approach for the blind nonlinear channel equalization problem.

2 RECURRENT CANONICAL PIECEWISE LINEAR ADAPTIVE EQUALIZER

RCPL network is first proposed in [10] based on the canonical piecewise linear network structure [4] and autoregressive moving average (ARMA) model. The basic idea involved in RCPL network is that RCPL network can partition the domain space where an ARMA model is used for approximation in each partitioned region. Since it is a global representation of piecewise linear structures, it requires a very small parameter space and allows for easy incorporation of statistical information into its structure. Because it also use feedback structures, its dynamic behavior is much powerful than that of feedforward structures. To better describe the input and output relationship of a dynamic process, a modified RCPL representation is given as follows:

Definition 1 (*Recurrent Canonical Piecewise-Linear Function*): A function f: $D_1 \times D_2 \times I \rightarrow Q$ with sample space $D_1 \subset R^N$, $D_2 \subset R^r$, index set I, and compact subset $Q \subset R^M$ is said to be a RCPL function if it can be expressed by the global representation:

$$f(\mathbf{x}(n), \mathbf{u}(n)) = \mathbf{a} + \mathbf{B}_1\mathbf{x}(n) + \mathbf{B}_2\mathbf{u}(n) \tag{1}$$

$$x_k(n) = a_k + \mathbf{b}_{1k}\mathbf{x}(n-1) + \mathbf{b}_{2k} f(\mathbf{x}(n-1), \mathbf{u}(n-1)) + \mathbf{b}_{3k}\mathbf{u}(n) + \sum_{i=1}^{\tau} c_{ki} |\langle \boldsymbol{\alpha}_{1ki}, \mathbf{x}(n-1)\rangle + \langle \boldsymbol{\alpha}_{2ki}, f(\mathbf{x}(n-1), \mathbf{u}(n-1))\rangle + \langle \boldsymbol{\alpha}_{3ki}, \mathbf{u}(n)\rangle + \beta_{ki}| \tag{2}$$

where, $\mathbf{x}, \in R^N$, $\mathbf{u} \in R^r$, $\mathbf{a} \in R^M$, $\mathbf{B}_1 \in R^{M\times N}$, $\mathbf{B}_2 \in R^{M\times r}$, $\mathbf{b}_{1k}, \boldsymbol{\alpha}_{1ki} \in R^{1\times N}$, $\mathbf{b}_{2k}, \boldsymbol{\alpha}_{2ki} \in R^{1\times M}$, $\mathbf{b}_{3k}, \boldsymbol{\alpha}_{3ki} \in R^{1\times r}$, $a_k, c_{ki}, \beta_{ki} \in R$ and $k = 1, 2, \cdots, N$. We refer to the structure defined by (1) and (2) as a *recurrent canonical piecewise linear network.*

The dynamics of RCPL equalizer can be constructed by the following set of equations based on the above definition:

$$z_k(n) = \omega_{k0}(n)\hat{x}(n-1) + \sum_{i=1}^{M} \omega_{ki}(n)\hat{x}_i(n-1) + \sum_{i=1}^{N} \omega_{ki+M}(n) y(n-i+1) \tag{3}$$

$$\hat{x}_k(n) = f_k(z_k(n)) \quad k = 1, 2, \cdots, M. \tag{4}$$

$$\hat{x}(n) = \sum_{i=1}^{M} \omega_{0i}(n)\hat{x}_i(n) + \sum_{i=1}^{N} \omega_{0i+M}(n) y(n-i+1) \tag{5}$$

where, $y(n)$ is the observed channel output corresponding to the transmitted signal $x(n)$ which takes values from a finite set $\mathcal{S}$, $\hat{x}(n)$ is the output of a unit trained to approximate $x(n)$, and $f_k(\cdot)$ is a piecewise-linear function.

If we choose the functions $f_k(\cdot)$ as

$$f_k(z_k(n)) = |z_k(n)+1| - |z_k(n)-1| \quad k = 1, 2, \cdots, M$$

and the cost function as the expectation of squared error as

$$J(n) = E\left\{e^2(n)\right\} = E\left\{(x(n) - \hat{x}(n))^2\right\},$$

then, a learning algorithm can be obtained by steepest descent minimization of $J(n)$:

$$\mathbf{w}_0(n+1) = \mathbf{w}_0(n) + \mu_1(n)e(n)\mathbf{x}(n)$$

$$\mathbf{w}_k(n+1) = \mathbf{w}_k(n) + \mu_2(n)e(n)\omega_{0k}v_k(n)\bar{\mathbf{x}}(n-1)$$

where $\bar{\mathbf{x}}^T(n-1) = [\hat{x}(n-1), \mathbf{x}(n-1)]$, $\mathbf{x}^T(n) = [\hat{x}(n), \hat{x}_1(n), \cdots \hat{x}_M(n), y(n), \cdots, y(n-N+1)]$, and $\mathbf{w}_j(n) = [\omega_{j0}, \omega_{j2}, \cdots, \omega_{jN+M}]$, $j = 0, 1, \cdots, M$. The above algorithm converges if the following conditions are satisfied:

$$0 < \mu_1(n) < \frac{2}{\lambda_1(n)}, \quad 0 < \mu_2(n) < \frac{2}{\lambda_2(n)}$$

Here,

$$v_k(n) = \mathrm{sgn}(z_k(n)+1) - \mathrm{sgn}(z_k(n)-1),$$

$$\mathbf{v}^T(n) = [v_1(n), \cdots, v_k(n)],$$

$$\mathbf{C}(n) = \mathrm{diag}(\omega_{01}, \cdots, \omega_{01}, \omega_{0M}, \cdots, \omega_{0M}).$$

$\lambda_1(n)$, $\lambda_2(n)$ are the maximum eigenvalues of matrices $E\left\{\mathbf{x}(n)\mathbf{x}^T(n)\right\}$, $\mathbf{C}(n)E\left\{\mathbf{x}(n)\mathbf{v}(n)\mathbf{v}^T(n)\mathbf{x}^T(n)\right\}\mathbf{C}^T(n)$ respectively.

Figure 1 compares the performance of the RCPL equalizer with that of the multilayer layer perceptron equalizer for a 16-PAM signal transmission over the multipath channel

$$G(z) = 1 + 0.5z^{-1} + 0.4z^{-2} + 0.02(1 + 0.5z^{-1} + 0.4z^{-2})^2$$

where RCPL equalizer has only 5 nodes and the MLP equalizer has 2 hidden layers with 11 nodes in each layer. As seen in this example, performance of RCPL equalizer is much superior to that of the MLP equalizer.

3 BLIND EQUALIZATION USING PARTIAL LIKELIHOOD ESTIMATION

In [1] and [2], we introduce a distribution learning framework for real-time signal processing with nonlinear structures (probability models) based on partial likelihood (PL) theory [5], [13]. PL is a particularly suitable formulation for real-time signal processing which most of the time requires on-line processing of dependent observations. We have introduced a general nonlinear network conditional probability model, and for this model, established a key information-theoretic connection, the equivalence of maximum PL estimation and accumulated relative entropy (ARE) minimization [1]. This result can be regarded as the extension of the ML and minimum ARE equivalence for i.i.d. data [12] to the general case of dependent observations. While providing the theoretical foundation for statistical analysis of maximum PL estimation, this connection can also be used to derive a new class of real-time signal processing algorithms based on information-theoretic alternating projections [3], [14]. In this paper, the maximum PL estimation is applied to blind equalization of binary communications channel and the conditional probability model is constructed based on the RCPL model (3), (4), (5) and statistical information of the channel inputs.

Assume that the only available information is channel observations: $y(n), y(n-1), \cdots, y(0)$, and the statistics of channel input $E[x(n)^j], j = 1, \cdots, 4$. Let $p_w(x(n)|\mathcal{F}_n)$ be the estimated probability mass function (pmf) of the input sequence $x(n)$, where $\mathcal{F}_n$ is the σ-field generated by events $[y(n), y(n-1), \cdots, y(1), y(0)]$. By [1], the equalization problem can be viewed as a distribution learning problem by maximizing partial log-likelihood function , i.e., $\max_w \sum_{i=1}^{i=n} \ln p_w(x(i)|\mathcal{F}_i)$. The true channel input $x(i)$ is not known, but it can be shown that the maximization of partial likelihood is equivalent to the maximization of quasi-partial log-likelihood function $\sum_{i=1}^{i=n} \ln p_w(\bar{x}_i|\mathcal{F}_i)$, $\bar{x}_i \in \mathcal{S}$ with respect to the ω and $\bar{x}_i \in \mathcal{S}$. Thus, By using this conclusion, the blind equalization algorithm is given as follows:

- Start with an initial estimate of ω
- Maximize quasi-partial log-likelihood function with respect to $\bar{x}_i$
- Maximize quasi-partial log-likelihood function with respect to ω based on the updated $\bar{x}_i$
- Repeat these steps until the algorithm converges

For the binary communication channel, $\mathcal{S} = \{-1, 1\}$, we choose

$$p_w(\bar{x}_n|\mathcal{F}_n) = \begin{cases} \frac{e^{-J(\bar{x}_n=1)}}{e^{-J(\bar{x}_n=1)}+e^{-J(\bar{x}_n=-1)}} & \text{if } \bar{x}=1 \\ 1-\frac{e^{-J(\bar{x}_n=1)}}{e^{-J(\bar{x}_n=1)}+e^{-J(\bar{x}_n=-1)}} & \text{if } \bar{x}=-1 \end{cases}$$

where

$$J(\bar{x}_n) = \sum_{j=1}^{4} \pi_j (E[\hat{x}_n^j] - E[x(n)^j])^2$$

$$E[\hat{x}_n^j] = \frac{1}{n}((n-1)E[\hat{x}_n^j] + \hat{x}_n^j), \quad \hat{x}_n^j = (\bar{x}_n^j + \hat{x}(n))/2$$

$\hat{x}(n)$ is the output of RCPL equalizer, and π_j are positive constants. Our blind algorithm and CMA algorithm [?] are tested for equalization of the nonlinear channel

$$G(z) = 1 + 0.7z^{-1} + 0.15(1 + 0.7z^{-1})^2$$

$$+0.1(1 + 0.7z^{-1})^3 + 0.05(1 + 0.7z^{-1})^4$$

Even for this relatively simple communication channel, the CMA based equalizer exhibits a very poor performance. In Figure 2, we plot the BER curves for both equalizers. The results show that RCPL based blind equalizer outperforms the CMA equalizer by orders of magnitude. If we choose the test channel as $G(z) = 1 + 0.7z^{-1}$, Figure 3 shows that RCPL blind equalizer and CMA algorithm have comparable performance.

4 CONCLUSIONS

A recurrent canonical piecewise linear equalizer has been proposed based on the RCPL network. The RCPL formulation provides a nice framework for the analysis of learning behavior of nonlinear recurrent adaptive structures and for choosing networks of appropriate complexity based on CPL approximation techniques. The simulations of 16-PAM channel equalization demonstrate the superior performance of RCPL adaptive equalizer. A novel blind algorithm is presented by combining partial likelihood estimation and RCPL network for the binary communication channel. The simulations demonstrate that RCPL based equalizer outperforms the CMA equalizer by orders of magnitude when equalizing nonlinear channel and they perform similarly in linear channel equalization.

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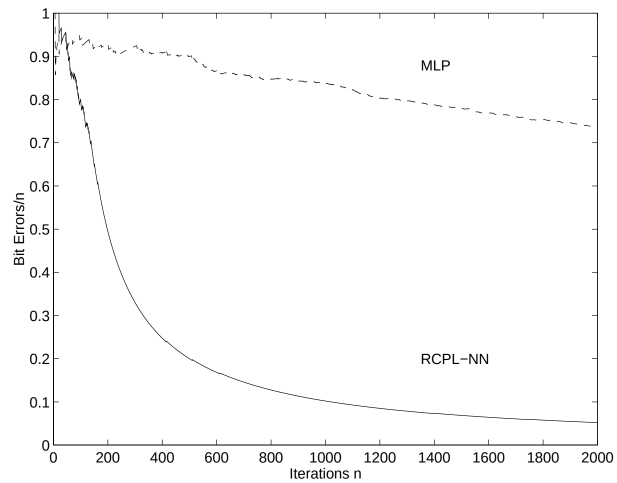

Figure 1: 16-PAM channel equalization

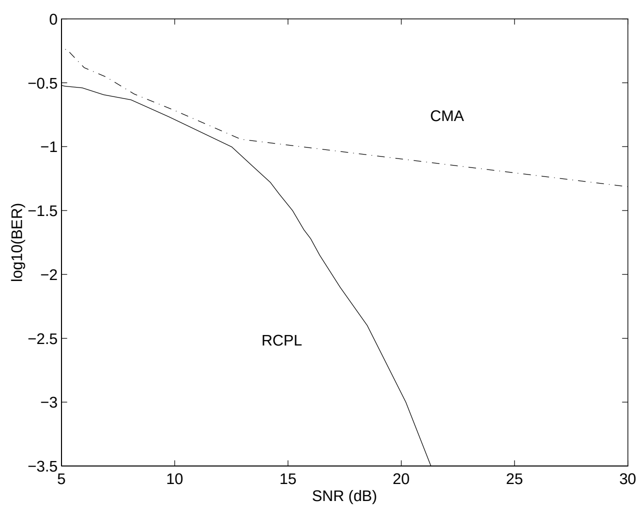

Figure 2: 2-PAM Nonlinear Blind Equalization

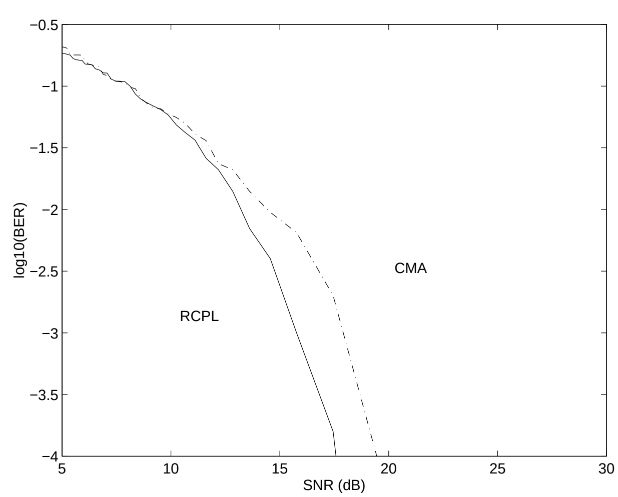

Figure 3: 2-PAM linear Blind Equalization