Second Order Blind Identification of Convolutive Mixtures with Temporally Correlated Sources: a Subspace Based Approach

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ABSTRACT

This contribution addresses the blind identification of Multiple Input Multiple Output (MIMO) linear FIR systems having a number of inputs less than the number of outputs. Recent publications have proposed an efficient second order identification method in the Single Input Multiple Output (SIMO) case. Based on a subspace analysis, it allows a perfect recovery of the system parameters and excitation in a noise free environment. In this paper we indicate how to extend the original subspace based approach to the general MIMO case.

1 INTRODUCTION

The problem of MIMO transfer functions identification typically appears in digital communications where several signals sharing the same frequency channel are subject to multipath propagation. This phenomenon usually leads to a considerable signal contamination requiring channel aquisition and further signal extraction. On the contrary to the classical approaches [1]-[3], some recent contributions proposed second-order methods of channel evaluation based on the output observation only [4]-[6]. This family of estimators is mainly represented by the linear prediction [7]-[9] and subspace based [10, 11] techniques originally proposed for the SIMO identification. A comparative study of these approaches in the SIMO case shows the linear prediction quite robust though less performant especially at high signal-to-noise ratio levels. On the other hand, the subspace based technique possesses a serious advantage since it does not require the contributing source signals to be *temporally uncorrelated* which is the essential condition for the linear prediction technique. Such a situation obviously motivates some development of the subspace based technique for the MIMO convolutional systems identification. A certain advance in this direction has been done in [11] where the authors discuss a special case of MIMO system identification. In particular, *a consistent MIMO convolutive channel identification up to a constant instantaneous mixture matrix is possible if all contributing sources have the same channel duration.* Under such a specific condition the global deconvolution problem can be reduced to the classical instantaneous mixture separation. The latter can be accomplished on the basis of HOS techniques [12]-[16] in the most general case. However the mentioned restriction on the propagation delays can be hardly ensured in a realistic environment.

In this context we propose here more detailed study of the subspace based techniques. The presented results originate from the relationships between the *rational space* of the MIMO transfer function and the signal subspace of the output spatio-temporal observation. Further analysis allows to recast the initial problem within the separation of convolutive mixtures having a particular structure of the transfer functions. This latter can be converted to a set of purely instantaneous mixtures.

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2 DATA MODEL AND HYPOTHESES

Let $\{x(t)\}_{t\in\mathbb{Z}}$ be an M-variate output process of a FIR system having overall duration L, driven by the m-variate process $\{s(t)\}_{t\in\mathbb{Z}}$:

$$x(t) = [H(z)]\, s(t), \qquad t \in \mathbb{Z}, \tag{1}$$

where $H(z) \triangleq \sum_{\tau=0}^{L} \mathbf{H}(\tau)\, z^{-\tau}$ is an $M \times m$ transfer function $H(z) = [H_1(z), \ldots, H_m(z)]$ with the column degrees $\deg(\, H_k(z)\,) = L_k$ equal to the relative propagation delay of the k-th source signal. Without loss of generality we suppose that $L_1 \leq \ldots \leq L_m = L$ and consider the blind identification of the MIMO transfer function $H(z) = [H_1(z), \ldots, H_m(z)]$ and further extraction of input $\{s(t)\}_{t\in\mathbb{Z}}$ from the *noise free observation* (1). In this paper we assume that:

H1 The number of inputs m is strictly less than the number of outputs M.

H2 The emitted sequences $\{s_k(t)\}_{t\in\mathbb{Z}}$, $k = 1, \ldots, m$, are statistically independent non purely harmonic processes†.

Most of the following results are based on the spatio-temporal properties of the observation process. We therefore define the finite order N *spatio-temporal observation* $X_N(t) \triangleq [x(t)^T, \ldots, x(t-N)^T]^T$. Now the equation (1) can be rewritten in the algebraic form

$$X_N(t) = \mathcal{T}_N(H)\, S_N(t), \qquad t \in \mathbb{Z}, \tag{2}$$

†Notice that $\{s_k(t)\}_{t\in\mathbb{Z}}$ may be temporally correlated.

where $S_N(t) \triangleq [s(t)^T, \ldots, s(t-N-L)^T]^T$ and $\mathcal{T}_N(H)$ is a *generalized Sylvester matrix* [17] associated with the polynomial $H(z)$:

$$\mathcal{T}_N(H) \triangleq \begin{bmatrix} H(0) & \ldots & H(L) & 0 & 0 \\ 0 & \ddots & & \ddots & 0 \\ 0 & 0 & H(0) & \ldots & H(L) \end{bmatrix}$$

i.e. a block-Toeplitz matrix having $N+1$ vertical and $N+L+1$ horizontal $M \times m$ blocks:

3 IDENTIFIABILITY

The pioneering contributions concerning SIMO analysis [6] show that second order identifiability requires certain channel disparity *i.e.* that the entries of the $M \times 1$ vector $H(z)$ have no common zeros. This condition admits an extension for the MIMO case known [18] as *irreducibility* of polynomial matrix:

H3 The polynomial matrix $H(z)$ is irreducible if the greatest common divisor of its $m \times m$ minors is 1 *i.e.* $\text{rank}(\ H(z)\ ) = m$ for all $z \in \mathbb{C}$.

Similarily to the SIMO case, this property plays an important role in source signals extraction. In particular, it allows to construct a FIR zero-forcing equalizer (ZFE) $E_H(z)$ of degree N providing the channel inversion: $s(t) = [E_H(z)]\, x(t)$. The following result is due to the generalized *Bezout identity*, [18].

Lemma 1 *There exists an $m \times M$ polynomial matrix $E_H(z)$ of finite order N such that $E_H(z)\, H(z) = \mathrm{I}_m$ if and only if* **(H3)** *holds.*

An upper bound of minimum possible N is also available for mono-source channel equalization. However the answer is not trivial in the MIMO context. In the general case $H(z)$ should satisfy an additional constraint.

H4 The polynomial matrix $H(z)$ is *column reduced i.e.* $\text{rank}(\ [\mathbf{H}_1(L_1), \ldots,\ \mathbf{H}_m(L_m)]\ ) = m$.

One can show [19] that under the conditions **(H3)** and **(H4)** there exists a ZFE of order no more than $N = \sum_{k=1}^{m} L_k$. On the other hand, these hypotheses are necessary and sufficient for the subspace based identification presented later in this paper. As a matter of fact, such a sophisticated mathematical properties have not received any clear physical equivalent. However a certain interpretation has been recently given [20]. Let us suppose that the set $\{\mathbf{H}_k(\tau)\}$ of all propagation modes is a realization of some propability law such that

Property 1 *For any subset $\{\mathbf{H}_{k_p}(\tau_p)\}_{p=1}^{m+1}$ of $m+1$ modes,* $\text{rank}\left([\mathbf{H}_{k_1}(\tau_1), \ldots, \mathbf{H}_{k_{m+1}}(\tau_{m+1})] \right) = m+1$ *with probability 1.*

Taking into account that $M \geq m+1$, this property clearly holds under the typical spatial diversity assumption. We have now the following result.

Theorem 1 *Let the modes of $H(z)$ be driven from some probability law verifying Property 1. Then* **(H3)** *and* **(H4)** *hold with probability 1.*

4 IDENTIFICATION APPROACH

We denote here by $\mathcal{S}(z)$ the column space of the polynomial matrix $H(z)$ *i.e.* $\mathcal{S}(z) \triangleq \text{span}\{\ H(z)\ \}$. Such a set can be treated as subspace of the M-dimensional *rational space* defined as the linear space over the field of rational fractions *i.e.* the space of M-dimensional vectors with rational entries. Since $\text{rank}(\ H(z)\ ) \leq m$ with the equality in some points under certain mild assumptions (*e.g.* $\text{rank}(\ \mathbf{H}(0)\ ) = m$), $\mathcal{S}(z)$ is a *rational subspace* of dimension m. As a matter of fact, the SIMO subspace estimator [10] originates from the relation between the range space of the spatio-temporal observation and the rational subspace uniquely generated by vector $H(z)$ under the disparity condition. In the most general case we can identify $\mathcal{S}(z)$ by reconstructing one of its arbitrary basis. Looking for some kind of minimal description, we choose here a *minimal polynomial basis* (MPB), see [21], *i.e.* an $M \times m$ polynomial matrix $B(z)$ with column degrees $\nu_1, \ldots, \nu_m$ having the minimal *channel order* $\nu \triangleq \sum_{k=1}^{m} \nu_k$. According to [21], the set of degrees $\{\nu_k\}_{k=1}^{m}$ is an invariant characteristic of the rational subspace. The same contribution provides the following criterion:

Lemma 2 *$B(z)$ is a MPB of $\mathcal{S}(z)$ if and only if* **(H3)** *and* **(H4)** *hold.*

As it follows from theorem 1 and lemma 2, $H(z)$ is some MPB of $\mathcal{S}(z)$ almost surely in typical applications and therefore $\nu_k = L_k,\ k = 1, \ldots, m$. We now indicate how to identify (estimate) a MPB of $\mathcal{S}(z)$. Let us define $\mathcal{S}^{\perp}(z)$ the *dual space* of $\mathcal{S}(z)$ *i.e.* the rational space of $1 \times M$ vectors: $\mathcal{S}^{\perp}(z) \triangleq \{g(z) \mid g(z)\, H(z) = 0\}$. This set of vectors forms a rational space of dimension $M-m$, we denote its degrees $\nu_1^{\perp}, \ldots, \nu_{M-m}^{\perp}$ and order $\nu^{\perp} = \sum_{k=1}^{M-m} \nu_k^{\perp}$. In fact, $\mathcal{S}^{\perp}(z)$ is uniquely associated with $\mathcal{S}(z)$, [21]. Consequently any $M \times m$ polynomial matrix $B(z) = \sum_{\tau=0}^{L} \mathbf{B}(\tau)\, z^{-\tau}$ of rank m is a basis of $\mathcal{S}(z)$ if and only if for any basis $g_1(z), \ldots, g_{M-m}(z)$ of $\mathcal{S}^{\perp}(z)$ we have $g_k(z)\, B(z) = 0,\ k = 1, \ldots, M-m$. This polynomial equation admits an algebraic equivalent due to the special structure of the Sylvester matrix. Let $g(z) = \sum_{\tau=0}^{N} \mathbf{g}(\tau)\, z^{-\tau}$ be an $1 \times M$ polynomial and the associated vector $\mathbf{g} \triangleq [\mathbf{g}(0), \ldots, \mathbf{g}(N)]$. Then,

$$g(z)\, B(z) = 0 \quad \Leftrightarrow \quad \mathbf{g}\, \mathcal{T}_N(B) = 0. \tag{3}$$

This duality allows to conclude that the column space of $B(z)$ lies in $\mathcal{S}(z)$ *if and only if* any set $\{\mathbf{g}_k\}_{k=1}^{M-m}$ associated with a basis of $\mathcal{S}^{\perp}(z)$, verifies $\mathbf{g}_k\, \mathcal{T}_N(B) = 0$. A set of vectors $\{\mathbf{g}_k\}_{k=1}^{M-m}$ can be also determined using the relationship (3) for the true channel: $\mathbf{g}_k\, \mathcal{T}_N(H) = 0$. Furthermore, according to (2) all vectors $\mathbf{g}_k$ satisfying this equality belong to the subspace orthogonal to the span of the noise-free observation $\{X_N(t)\}_{t \in \mathbb{Z}}$ of order $N \geq \deg(\ g_k(z)\ ),\ k = 1, \ldots, M-m$. Suppose that we can choose N not less than the maximum of degrees $\nu_k^{\perp}$. Then the orthogonal complement to the signal subspace of $\{X_N(t)\}_{t \in \mathbb{Z}}$ contains the set $\{\mathbf{g}_k\}_{k=1}^{M-m}$ associated with some basis of $\mathcal{S}^{\perp}(z)$. Consequently any matrix $\mathbf{G}_N$ with lines spanning the orthogonal (noise) subspace of $\{X_N(t)\}_{t\in\mathbb{Z}}$ describes the rational subspace $\mathcal{S}(z)$ in the sense that $\mathbf{G}_N\,\mathcal{T}_N(B) = 0$ implies span$\{\,B(z)\,\} \in \mathcal{S}(z)$. We also need that $B(z)$ is a MPB of $\mathcal{S}(z)$ *i.e.* that **(i)** $\deg(\,B_k(z)\,) = L_k$ and **(ii)** the maximum column rank of $B(z)$ is m, this latter provided by rank$(\,\mathbf{B}(0)\,) = m$. To meet both **(i)** and **(ii)**, several methods were discussed [11] in the case $L_1 = \ldots = L_m$. Meanwhile, these conditions can be satisfied in the general case if the upper $m \times m$ block of $\mathbf{B}(0)$ is a lower triangular matrix with non-zero diagonal elements. The above discussions lead to the following result. Let us denote a set of $M \times m$ polynomial matrices verifying $\mathcal{B} \triangleq \{\,B(z)\mid \mathbf{B}_{pk}(0) = 0,\ p<k;\ \mathbf{B}_{kk}(0) = 1,\ \mathbf{B}_{pk}(\tau) = 0\ \forall\ p, \tau > L_k,\ 1 \le k \le m\,\}$.

Theorem 2 *There exists $B(z) \in \mathcal{B}$ a MPB of $\mathcal{S}(z)$. Alternatively, any $B(z) \in \mathcal{B}$ is a MPB of $\mathcal{S}(z)$ if and only if $\mathbf{G}_N\,\mathcal{T}_N(B) = 0$.*

The orthogonal subspace matrix $\mathbf{G}_N$ can be identified or at least consistently estimated in the noisy case from any empirical counterpart of $\mathbf{R}_x = \mathbb{E}\left\{X_N(t)\,X_N(t)^H\right\}$. Such an estimate $\hat{\mathbf{G}}_N$ is usually obtained via the eigendecomposition of the estimate $\hat{\mathbf{R}}_x$. According to theorem 2, a consistent estimate $\hat{B}(z)$ of some MPB of $\mathcal{S}(z)$ can be found as the solution of the following problem:

$$\hat{B}(z) = \arg\min_{\mathcal{B}} \|\,\hat{\mathbf{G}}_N\,\mathcal{T}_N(B)\,\|_F. \tag{4}$$

It is easy see that (4) is a quadratic minimization under the linear constraints $\mathcal{B}$. This classical problem, [22], yields an explicit solution. Note that the empirical column space coincides with the true one in the noise-free case. It means that *one can perfectly identify some MPB of $\mathcal{S}(z)$ from a finite observation sample.* The complete estimation procedure requires the knowledge of minimum admissible N as well as the dimension of the signal subspace. In fact the degrees of $\mathcal{S}^\perp(z)$ depend non-trivially upon the parameters of $H(z)$. However a kind of the *upper bound* for minimum value N can be found due to the equality of orders for dual spaces *i.e.* $\nu = \nu^\perp$, see [21]. We obviously have $\nu_k^\perp \le \nu^\perp$ and therefore a sufficient condition $N \ge \nu = \sum_{k=1}^m L_k$. Now the signal subspace dimension coincides with the column space of $\mathcal{T}_N(H)$. According to [21], this matrix has rank equal to $(N+1)m + \nu$ under **(H3)** and **(H4)**. Both N and signal subspace dimension can be calculated from the values $L_1, \ldots, L_m$. However imperfect knowledge of these quantities leads to the general failure of the estimator (4).

Our next task is to recover the MPB $H(z)$ from some arbitrary MPB of $\mathcal{S}(z)$. We obviously need to describe the family of all MPB. Let $\mathcal{L}_1 < \ldots < \mathcal{L}_d$ be the different values of $\{L_k\}$ and $\mu_1, \ldots, \mu_d$ the numbers of equal degrees within each group ($L_1 = \ldots = L_{\mu_1} = \mathcal{L}_1$, etc).

Theorem 3 *Let $H(z)$ and $B(z)$ be some MPB of $\mathcal{S}(z)$. Then $H(z) = B(z)\,R(z)$, where $R(z)$ is an $m\times m$ polynomial matrix*

$$R(z) = \begin{bmatrix} R_{11} & R_{12}(z) & \ldots & R_{1d}(z) \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & R_{d-1\,d}(z) \\ 0 & \ldots & 0 & R_{dd} \end{bmatrix},$$

$R_{pq}(z)$ are $\mu_p \times \mu_q$ polynomial matrices of degrees $\mathcal{L}_q - \mathcal{L}_p$, $p \le q$, and R_{pp} are constant non-singular.

To proceed with the identification of $R(z)$, we propose first to "compensate" previously found factor $B(z)$. According to lemma 2, there exists a left inverse $E_B(z)$ of $B(z)$ of order N such that $E_B(z)\,B(z) = \mathrm{I}_m$ *i.e.* one has from (1) and theorem 3

$$v(t) = [E_B(z)]\,x(t) \ \Rightarrow\ v(t) = [R(z)]\,s(t), \quad t \in \mathbb{Z}, \tag{5}$$

where $\{v(t)\}_{t\in\mathbb{Z}}$ is m-variate series. A staightforward approach to retrieve $\{v(t)\}_{t\in\mathbb{Z}}$ can be deduced via the algebraic equation for $\{x(t)\}_{t\in\mathbb{Z}}$ and $\{v(t)\}_{t\in\mathbb{Z}}$ similar to (2): $X_N(t) = \mathcal{T}_N(B)\,V_N(t)$, $V_N(t) \triangleq [v(t)^T, \ldots, v(t-N-L)^T]^T$. As one can see, $v(t)$ may be obtained as m upper entries of $\mathcal{T}_N(B)^{\#}\,X_N(t)$, where $(^{\#})$ stands for the pseudo-inverse.

Let us study the case of equal degrees: $L_1, \ldots, L_m$. We have $d = 1$ *i.e.* $\mu_1 = m$ and therefore

$$v(t) = R_{11}\,s(t), \qquad t \in \mathbb{Z} \tag{6}$$

is an instantaneous mixture of the source signals. Due to the error-free identification of $B(z)$, *perfect reduction to the instantaneous MIMO problem is possible in the case of equal degrees from a finite noise-free sample.*

In the most general case, we still need to invert the convolutive mixture (5). However the specific structure of $R(z)$ allows to construct a simple FIR inverse.

Lemma 3 *There exists a unique structured $m\times m$ polynomial matrix*

$$Q(z) = \begin{bmatrix} \mathrm{I}_{\mu_1} & Q_{12}(z) & \ldots & Q_{1d}(z) \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & Q_{d-1\,d}(z) \\ 0 & \ldots & 0 & \mathrm{I}_{\mu_m} \end{bmatrix}, \tag{7}$$

$Q_{pq}(z)$ are $\mu_p \times \mu_q$ polynomial matrices of the degrees $\mathcal{L}_q - \mathcal{L}_p$, $p \le q$, such that

$$Q(z)\,R(z) = R \quad \textit{with} \quad R = \begin{bmatrix} R_{11} & 0 & \ldots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & R_{dd} \end{bmatrix}.$$

Further application of the filter $Q(z)$ to the pre-filtered series $\{v(t)\}_{t\in\mathbb{Z}}$ provides us the output signal

$$u(t) = [Q(z)]\,v(t) \ \Rightarrow\ u(t) = R\,s(t), \quad t \in \mathbb{Z}. \tag{8}$$

Identification of $Q(z)$ can be accomplished within the framework of second order statistics as shown in the following section. Notice that due to the block-diagonal structure of R the output series $\{u(t)\}_{t\in\mathbb{Z}}$ is a set of d non-overlapping instantaneous mixtures each including the source signals having the same channel degree. Except of the equal degrees case, each instantaneous mixture contains less than m components which simplifies further source separation. As already mentioned, one can apply various HOS separation techniques to complete the identification procedure.

Finalizing this section we examine another particular case: $L_1 < \ldots < L_m$ *i.e.* when all degrees are different. It is easy to check that $d = m$ and $\mu_1, \ldots, \mu_m = 1$. Consequently, R is a diagonal matrix. This obviously means that the entries of $u(t)$ are somehow scaled source signals $s(t)$ *i.e. convolutive MIMO mixtures can be separated via the second order analysis in the case of strictly different degrees.*

4.1 Identification of $Q(z)$

The proposed approach is based on the statistical independence of different source signals. Observing the structure of $R(z)$, one can notice that the integral output power of signals $v(t)$ is more than the power of $u(t)$ if there are some non-zero entries $R_{pq}(z)$. So the generic idea is to choose $Q(z)$ of the form (7) minimizing the overall output power.

Let $T = \deg(\ Q(z)\ )$, $F(z) = \sum_{\tau=0}^{T} \mathbf{F}(\tau)\, z^{-\tau}$ be an $m \times m$ polynomial filter having the structure (7) and $\{u_F(t)\}_{t\in\mathbb{Z}}$ an m-variate time series

$$u_F(t) = [F(z)]\, v(t) \qquad t \in \mathbb{Z}. \tag{9}$$

We also define the integral output power of $u_F(t)$ as $\mathcal{W}(F) \triangleq \mathbb{E}\left\{u_F(t)^H\, u_F(t)\right\}$.

Theorem 4 *Under the hypothesis* **(H2)**, *the minimum variance equation*

$$\mathcal{W}(\tilde{F}) = \min_{F} \mathcal{W}(F) \tag{10}$$

has a unique solution $\tilde{F}(z) = Q(z)$.

To explicit (10) let us write $u_F(t) = \mathbf{F}\, V_T(t)$ with $\mathbf{F} = [\mathbf{F}(0), \ldots, \mathbf{F}(T)]$. Now, $\mathcal{W}(F) = \mathrm{tr}\left(\mathbf{F}\,\mathbf{R}_v\,\mathbf{F}^H\right)$, where $\mathbf{R}_v \triangleq \mathbb{E}\left\{V_T(t)\, V_T(t)^H\right\}$. One can see that the criterion (10) is given by a certain quadratic form on the set of coefficients $\mathbf{F}$ while the structure restrictions (7) can be formulated as a set of linear constraints on $\mathbf{F}$. Therefore the explicit expression for $Q(z)$ can be conventionally found as a function of $\mathbf{R}_v$. Furthermore, a consistent estimate of $Q(z)$ can be calculated from the empirical counterpart of $\mathbf{R}_v$, this latter being available via $\hat{\mathbf{R}}_x$ according to (5).

SUMMARY

As shown in this paper, the second order subspace based analysis allows to transform a convolutive mixture of independent signals to a group of instantaneous mixtures. The direct extension of the traditional approach known in a single source case requires some further linear filtering in the most general case. The discussed technique can be essentially better than the linear prediction for certain applications since it is tolerant to the temporal correlation of the source signals. Various particular cases presented in this paper underline a crucial meaning of channel degrees structure for identifiability and estimation complexity. On the other hand, the discussed technique is sensitive to the imperfect knowledge of these degrees. However this drawback may be overcome in digital communications using short reference sequences *i.e.* combinig classisal techniques with the presented blind identification approach.

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