A DESIGN METHOD FOR OVERSAMPLED PARAUNITARY DFT FILTER BANKS USING HOUSEHOLDER FACTORIZATION

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ABSTRACT

In this work, we propose a design method for oversampled FIR DFT filter banks which have the paraunitary property, where the number of channel M is the multiple of decimation ratio D and the filter length is the multiple of M. Our proposed method is based on Householder factorization, which can keep the perfect reconstruction condition and the paraunitary property of filter banks in optimization process. In addition, we examine the linear phase property for oversampled DFT filter banks, and the design method of oversampled linear phase DFT filter banks. In order to show the effectiveness of our method, we give some design examples.

1 INTRODUCTION

DFT filter banks[1] are known as efficient design methods for filter banks, because both analysis and synthesis filters can be given by designing only one analysis and one synthesis filter, respectively. However, there are no significant FIR filters satisfying perfect reconstruction(PR) condition for maximally decimated DFT filter banks[1]. In the case of Oversampled(OS) DFT filter banks[2, 3], there exist FIR filters which satisfy the PR condition. As previous works, some design conditions for OS DFT filter banks have been shown in [2, 3]. In particular, the article [3] shows a design method for OS DFT filter banks and applying them to subband adaptive filtering[3, 4, 5]. However, this conventional method have the constraint that filter length is imposed to be less than M. As a result, there is a problem that designed filters have the limitation of stopband attenuation.

On the other hand, it is known that maximally decimated filter banks can be designed easily by using paraunitary property[1, 6], which is sufficient for the PR condition. In paraunitary filter banks, synthesis filters can be obtained as the time reversed coefficients of the counter part analysis filters. Therefore, paraunitary filter banks are given by designing only analysis filters. In addition, the paraunitary property can be kept in optimization process by using Householder factorization[1, 6]. Although the article [7] shows a design condition for OS DFT filter banks using paraunitary property, the general design method has not been addressed.

In order to solve the problem that designed filters have the limitation of stopband attenuation, we propose a design method for PR OS FIR DFT filter banks by using Householder factorization under a condition that M is a multiple of D. We refer them as OS paraunitary DFT filter banks. By using the proposed method, OS paraunitary DFT filter banks can be designed by controlling some parameter vectors for minimizeing some object function in optimization process. In addition, we also show a design method for OS paraunitary DFT filter banks with linear phases[1, 6] and that the number of design parameters can be reduced in this case. In order to show the effectiveness of our method, we give design some examples.

2 OVERSAMPLED FILTER BANKS[2,7]

In this section, we review OS filter banks based on polyphase representation. It is shown that a PR condition of OS DFT filter banks and the definition of OS paraunitary DFT filter banks.

Figure 1(a) shows a structure of an M-channel OS filter bank with decimation ratio D, which consists of M analysis filters $H_k(z)$ and M synthesis filters $F_k(z)$, $k = 0, 1, \cdots, M-1$. The boxes including $\downarrow D$ and $\uparrow D$ denote a down-sampler and an up-sampler with the factor D, respectively. When the reconstructed output sequence $\hat{X}(z)$ is identical to the input $X(z)$ except for the delay and scaling, the analysis-synthesis system is called perfect reconstruction(PR) filter banks.

2.1 PR condition based on polyphase representation

The structure shown in Fig.1(a) can always be expressed in terms of the polyphase matrices as shown in Fig.1(b), where $\boldsymbol{E}^D(z)$ and $\boldsymbol{R}^D(z)$ denote the $M \times D$ and $D \times M$ polyphase matrices, corresponding to analysis and synthesis banks, respectively. Let $\boldsymbol{H}(z)$ and $\boldsymbol{F}(z)$ be $M \times 1$ column vectors defined by

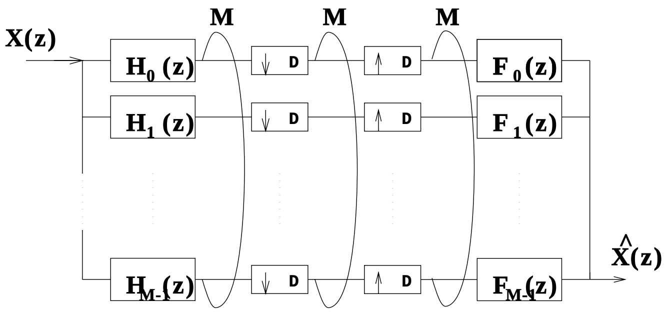

Figure 1(a) : OS filter bank

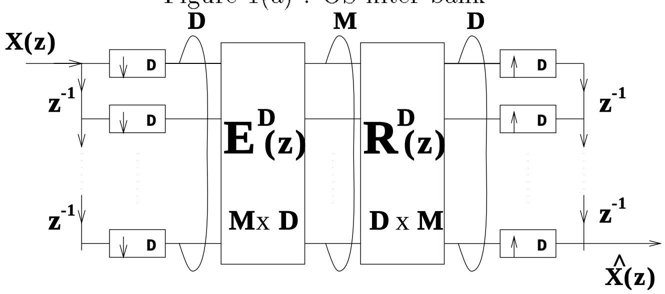

Figure 1(b) : Polyphase representation

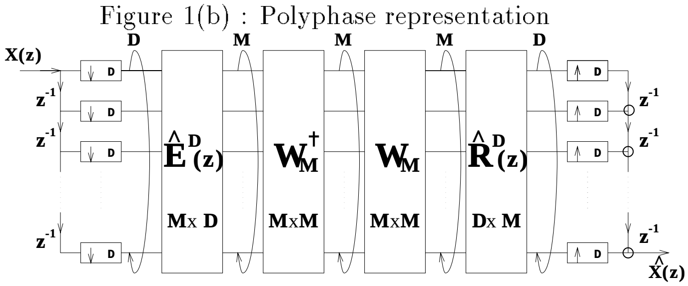

Figure 2 : OS DFT filter bank

$$\boldsymbol{H}(z) = [H_0(z), H_1(z), \cdots, H_{M-1}(z)]^T \quad (1)$$
$$\boldsymbol{F}(z) = [F_0(z), F_1(z), \cdots, F_{M-1}(z)]^T \quad (2)$$

The relations between $\boldsymbol{E}^D(z)$ and $\boldsymbol{H}(z)$ and between $\boldsymbol{R}^D(z)$ and $\boldsymbol{F}(z)$ are represented as

$$\boldsymbol{H}(z) = \boldsymbol{E}^D(z)\boldsymbol{z}(z) \quad (3)$$
$$\boldsymbol{F}(z) = z^{-(D-1)}\boldsymbol{z}^T(z^{-1})\boldsymbol{R}^D(z) \quad (4)$$

where $\boldsymbol{z}(z) = [1, z^{-1}, \cdots, z^{-(D-1)}]^T$. If $\boldsymbol{E}^D(z)$ and $\boldsymbol{R}^D(z)$ satisfy the condtion

$$\boldsymbol{R}^D(z)\boldsymbol{E}^D(z) = z^{-m}\boldsymbol{I}_D \quad (5)$$

for some integer m, where $\boldsymbol{I}_D$ is the $D \times D$ identity matrix, then the filter banks have the PR property.

2.2 OS DFT filter banks

Figure 2 shows a structure of an OS DFT filter bank. For DFT filter banks, the filters $H_k(z)$ and $F_k(z)$ in Fig. 1(a) are given by the prototype filters $H_0(z)$ and $F_0(z)$ as $H_k(z) = H_0(zW_M^k)$ and $F_k(z) = W_M^{-k}F_0(zW_M^k)$ with $W_M = \exp(-j\frac{2\pi}{M})$. Then, the polyphase matrices $\boldsymbol{E}^D(z)$ and $\boldsymbol{R}^D(z)$ can be expressed as $\boldsymbol{E}^D(z) = \boldsymbol{W}_M^{\dagger}\hat{\boldsymbol{E}}^D(z)$ and $\boldsymbol{R}^D(z) = \hat{\boldsymbol{R}}^D(z)\boldsymbol{W}_M$, respectively, where $\boldsymbol{W}_M$ is the $M \times M$ DFT matrix and '$\boldsymbol{W}_M^{\dagger}$' means transpose-conjugate[1] of $\boldsymbol{W}_M$. Since $\boldsymbol{W}$ is the orthogonal, the PR condition for OS DFT filter banks is reduced to the following condition.

$$\hat{\boldsymbol{R}}^D(z)\hat{\boldsymbol{E}}^D(z) = cz^{-m}\boldsymbol{I}_D \quad (6)$$

In Eq.(6), $\hat{\boldsymbol{E}}^D(z)$ and $\hat{\boldsymbol{R}}^D(z)$ are the $M \times D$ and $D \times M$ matrices given by

$$\hat{\boldsymbol{E}}^D(z) = \hat{\boldsymbol{I}} \cdot \boldsymbol{E}_P(z) \cdot \boldsymbol{Z}(z) \quad (7)$$
$$\hat{\boldsymbol{R}}^D(z) = \tilde{\boldsymbol{Z}}(z) \cdot \boldsymbol{R}_P(z) \cdot \hat{\boldsymbol{I}}^T \quad (8)$$

where $\tilde{\hat{\boldsymbol{Z}}}^D(z)$ is the paraconjugation[1] of $\hat{\boldsymbol{Z}}^D(z)$, N is the least common multiple of M and D, r_M and r_D are two integers satisfying $N = r_M M = r_D D$, '^{T}' means the transpose of the matrix and $\hat{\boldsymbol{I}}, \boldsymbol{Z}(z), \boldsymbol{E}_P(z)$, and $\boldsymbol{R}_P(z)$ are given by

$$\hat{\boldsymbol{I}} = \underbrace{[\boldsymbol{I}_M, \cdots, \boldsymbol{I}_M]}_{1\times r_M} \quad (9)$$
$$\boldsymbol{Z}(z) = \underbrace{[\boldsymbol{I}_D, z^{-1}\boldsymbol{I}_D, \cdots, z^{-(r_D-1)}\boldsymbol{I}_D]}_{1\times r_D}{}^T \quad (10)$$
$$\boldsymbol{E}_P(z) = diag[E_0(z^{r_D}), \cdots, E_{N-1}(z^{r_D})] \quad (11)$$
$$\boldsymbol{R}_P(z) = diag[R_0(z^{r_D}), \cdots, R_{N-1}(z^{r_D})] \quad (12)$$

where $E_l(z^{r_D})$ and $R_l(z^{r_D})$ are the polyphase elements of the prototype filters $H_0(z)$ and $F_0(z)$, respectively. Therefore, DFT filter banks are given by designing only the two filters $H_0(z)$ and $F_0(z)$.

2.3 OS paraunitary DFT filter banks

When an OS DFT filter bank satisfies the condition,

$$\tilde{\hat{\boldsymbol{E}}}^D(z)\hat{\boldsymbol{E}}^D(z) = cz^{-K}\boldsymbol{I}_D \quad (13)$$

the filter bank is called an OS paraunitary DFT filter bank, where c is a constant. From the property of paraunitary filter banks, the polyphase matrix $\hat{\boldsymbol{R}}^D(z)$ for synthesis bank can be determined as $\hat{\boldsymbol{R}}^D(z) = z^{-K}\tilde{\hat{\boldsymbol{E}}}^D(z)$, with some integer K. Thus, under this condition, the filter bank can be given by designing only one prototype filter $H_0(z)$, because $\tilde{\hat{\boldsymbol{E}}}^D(z)$ is given only by $H_0(z)$.

3 PROPOSED DESIGN METHOD

In this section, we show the proposed method for designing OS paraunitary DFT filter banks where M is a multiple of D, that is equivalent to $r_M = 1$. We are proposing to use Householder's factorization[1] for designing a prototype filter $H_0(z)$ satisfying the condition $\tilde{\hat{\boldsymbol{E}}}^D(z)\hat{\boldsymbol{E}}^D(z) = c\boldsymbol{I}_D$. The advantage of this approach is that the filter $H_0(z)$ can be designed by controlling some unit norm vectors[1], because $\hat{\boldsymbol{E}}^D(z)$ is guaranteed to keep the paraunitary property in the optimization process. In addition, we show a design method for OS linear phase paraunitary DFT filter banks, and consider a way to select initial parameters for the optimization.

3.1 Factorization of polyphase matrix

A polyphase matrix $\hat{\boldsymbol{E}}^D(z)$ in Eq.(7) can be rewritten when $r_M = 1$, as follows:

$$\hat{\boldsymbol{E}}^D(z) = [\boldsymbol{e}_0(z), \boldsymbol{e}_1(z), \cdots, \boldsymbol{e}_{r_D-1}]^T \quad (14)$$

$$\boldsymbol{e}_k(z) = z^{-k} \cdot diag[E_{Dk}(z), E_{Dk+1}(z), \cdots, E_{D(k+1)-1}(z)] \quad (15)$$

Paraunitary matrices factorized by Householder factorization should not include '0'-valued element. However $\hat{\boldsymbol{E}}^D(z)$ has some '0'-valued elements in Eqs. (14) and (15). Thus, in order to discard '0'-valued elements from $\hat{\boldsymbol{E}}^D(z)$, we decompose $\hat{\boldsymbol{E}}^D(z)$ into sub-matrices $\boldsymbol{p}_l(z)$,$l = 0, 1, \cdots, D-1$, as follows:

$$\underbrace{\boldsymbol{p}_l(z)}_{r_D \times 1} = \boldsymbol{S}_l \hat{\boldsymbol{E}}^D(z) \boldsymbol{T}_l(z) = [E_l(z), z^{-1}E_{l+D}(z), \cdots, z^{-(r_D-1)}E_{l+(r_D-1)D}(z)]^T \quad (16)$$

$$\underbrace{[\boldsymbol{S}_l]_{ij}}_{r_D \times M} = \begin{cases} 1 & (j = Di + l) \\ 0 & (\text{others}) \end{cases} \quad (17)$$

$$\underbrace{[\boldsymbol{T}_l]_{i0}}_{D \times 1} = \begin{cases} 1 & (i = l) \\ 0 & (\text{others}) \end{cases} \quad (18)$$

where $[\boldsymbol{S}_l]_{ij}$ presents the element of i-th row and j-th column of the matrix $\boldsymbol{S}_l$. If $\boldsymbol{p}_l(z)$ is paraunitary, $\hat{\boldsymbol{E}}^D(z)$ become a paraunitary matrix, because both $\boldsymbol{S}_l$ and $\boldsymbol{T}_l$ are paraunitary ones. Since there is the relation $\hat{\boldsymbol{E}}^D(z) = \sum_{l=0}^{D-1} \boldsymbol{S}_l^T \boldsymbol{p}_l(z) \boldsymbol{T}_l^T$, we can design $\hat{\boldsymbol{E}}^D(z)$ by controlling $\boldsymbol{p}_l(z)$ under the paraunitary constraint $\tilde{\boldsymbol{p}}_l(z)\boldsymbol{p}_l(z) = 1$.

3.2 Householder factorization

The $r_D \times 1$ paraunitary matrices $\boldsymbol{p}_l(z), l = 0, 1, \cdots D-1$, can be factorized into parameter matrices $\boldsymbol{V}_n(z)$ and $\boldsymbol{Q}$ by using Householder factorization , as follows:

$$\boldsymbol{p}_l(z) = \boldsymbol{V}_L(z)\boldsymbol{V}_{L-1}(z) \cdots \boldsymbol{V}_1(z)\boldsymbol{Q} \quad (19)$$

$$\boldsymbol{V}_n(z) = \boldsymbol{I} - \boldsymbol{v}_n\boldsymbol{v}_n^\dagger + z^{-1}\boldsymbol{v}_n\boldsymbol{v}_n^\dagger \quad (20)$$

$$\boldsymbol{Q} = \boldsymbol{I} - 2\boldsymbol{q}\boldsymbol{q}^\dagger \quad (21)$$

where L is the order of $\boldsymbol{p}_l(z)$ and both $\boldsymbol{v}_n$ and $\boldsymbol{q}$ are $r_D \times 1$ unit norm vectors[1, 6].

3.3 Linear Phase Property and Initial parameters

Here, a design method for OS DFT filter banks with linear phases is derived. If $\hat{\boldsymbol{E}}^D(z)$ satisfies,

$$\hat{\boldsymbol{E}}^D(z) = \boldsymbol{J}_M z^{-(L+r_D-1)} \hat{\boldsymbol{E}}^D(z^{-1}) \boldsymbol{J}_D, \quad (22)$$

$H_0(z)$ has a linear phase, where $\boldsymbol{J}_n$ is the $n \times n$ reversal matrix[1]. From Eq.(16), $\boldsymbol{p}_l(z)$ satisfies the condition in Eq.(22) as $\boldsymbol{p}_l(z) = \boldsymbol{J}_{r_D} z^{-(L+r_D-1)} \boldsymbol{p}_{D-l-1}(z^{-1})$. It is easily shown that there exist $\lceil \frac{D}{2} \rceil$ pairs of $\boldsymbol{p}_l(z)$ which satisfy this condition, where $\lceil x \rceil$ means the smallest integer which is bigger than x. We apply the condition to designing OS linear phase paraunitary DFT filter banks.

Next, let us consider how to select the initial parameters for the optimization. We propose to choose initial parameters as $\boldsymbol{v}_n = \boldsymbol{q} = [1, 0, \cdots, 0]^T$ which satisfy paraunitary and linear phase properties. Under these properties, the initial prototype filter $H_{init}(z)$ is expressed as

$$H_{init}(z) = \sum_{n=0}^{D-1} z^{-(n+(L-1)M-1)} \quad (23)$$

Using this filter, we can design a low pass filter. As a result, it is possible to use this initial prototype filter for designing OS DFT filter banks.

3.4 The number of parameters

Here, we investigate the number of parameters needed for optimization of $H_0(z)$. A prototype filter $H_0(z)$ which satisfies the PR condition can be designed by controlling vectors $\boldsymbol{v}_n$ and $\boldsymbol{q}$. Since each $\boldsymbol{v}_n$ and $\boldsymbol{q}$ contains $r_D - 1$ parameters[8], the total number of parameters for optimizing $H_0(z)$ is given by

$$P = D \times (r_D - 1)(L + 1). \quad (24)$$

If $H_0(z)$ has a linear phase, the total number of parameters can be reduced to

$$P_{lp} = \lceil \frac{D}{2} \rceil \times (r_D - 1)(L + 1). \quad (25)$$

4 Design Examples

We show design some examples using the proposed method. Although the proposed method is available for any kinds of object functions ,we choose two cases of objective functions as follows:

- Case 1 : $Minimize : max\{\left|H_0(\exp(j\omega))\right| : \omega = \frac{\pi}{D} + \delta, \frac{\pi}{D} + \delta + \frac{\pi}{512}, \cdots, \pi\}$
- Case 2 : $Minimize : \int_{\frac{\pi}{D}+\delta}^{\pi} \left|H_0(\exp(j\omega))\right|^2 d\omega$

where δ is the width of transition band region which we set $\frac{\pi}{2D}$. For Figs. 3 and 4, we choose $M = 32$, $D = 16$ and $L = 2$. In Fig 4, the filters have linear phases. As a result, we obtained the filters shown in Figs.3 and 4.

The initial filter is:

$$H_{init}(z) = \sum_{n=0}^{15} z^{-(n+31)} \quad (26)$$

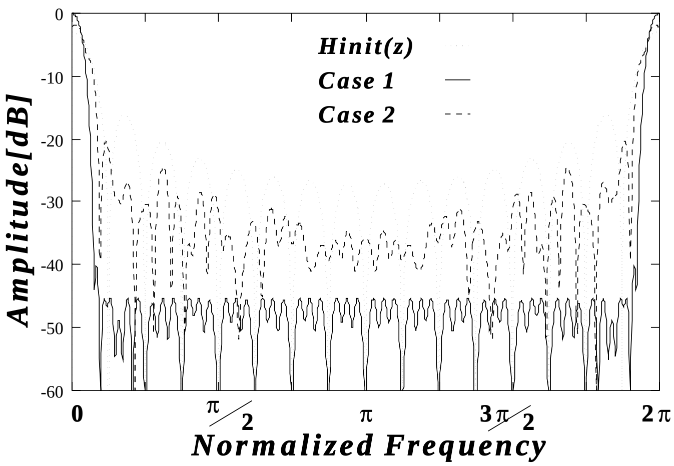

Figure 3 : Amplitude responses of
$H_0(z)$
for $M = 32, D = 16, L = 2$

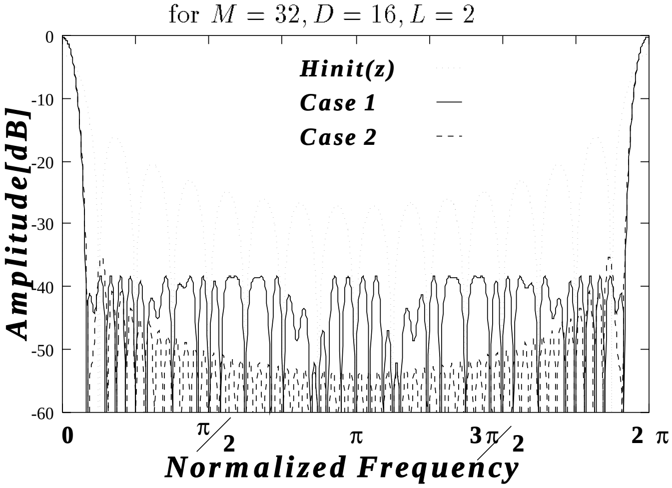

Figure 4 : Amplitude responses $H_0(z)$
with linear phase
for $M = 32, D = 16, L = 2$

The total number of parameters for optimization $H_0(z)$ is

$$P = 16 \times (2-1)(2+1) = 48 \qquad (27)$$

and if $H_0(z)$ has a linear phase, the total number of parameters can be reduced to

$$P_{lp} = \lceil \frac{16}{2} \rceil \times (2-1)(2+1) = 24 \qquad (28)$$

5 CONCLUSION

In this work, we proposed a design method for OS paraunitary DFT filter banks where the number of channel M is the multiple of decimation ratio D, and filter length is the multiple of M by using Householder factorization. Our proposed method can keep the perfect reconstruction condition and the paraunitary property of filter banks in the optimization process. For the optimization processes, we chose two minimizing object function. In addition, we also showed a design method for OS linear phase paraunitary DFT filter banks. In order to show effectiveness of our method, we showed some design examples.

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