A UNIFIED APPROACH TO ROBUST ADAPTIVE BEAMFORMING IN MOVING JAMMER ENVIRONMENT

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ABSTRACT

The performance of adaptive beamforming algorithms is known to degrade in rapidly moving jammer environments. This degradation occurs due to the jammer motion that may bring the jammers out of the sharp nulls of the adapted directional pattern. Below, we develop a unified approach allowing to make a wide class of adaptive array algorithms robust against possible jammer motion. This is achieved by means of artificial broadening of the null width in all jammer directions. Data-dependent sidelobe derivative constraints are used which do not require any *a priori* information about the jammers. The robust modifications of several well known adaptive array algorithms are formulated.

1. INTRODUCTION

The performance of adaptive arrays severely degrades if the weights are not able to adapt sufficiently fast to the changing (non-stationary) jamming situation or to the antenna platform motion. Fast adaptation has therefore be the aim of research in these cases. Moving jammers represent a serious problem, because for large antennas the directional pattern nulls are extremely sharp and jammers may soon move out of the nulls, i.e. high gain antennas are very sensitive to this type of non-stationarity.

Recently, a large number of robust adaptive beamforming methods has been studied. However, only the robustness against desired signal positioning errors was addressed. In this paper, another problem is considered. We assume that the desired signal direction is exactly known, whereas the relative angular motion of the jammers may be fast. In other words, we address another type of robust adaptive array – the robustness against possible jammer motion. We develop a unified framework allowing to incorporate this robustness property into various adaptive beamforming algorithms. The main idea is to broaden the width of the pattern nulls in the jammer directions. For this purpose, derivative constraints are used, which do not require any *a priori* information about the jammers. Similar constraints have been used in [1], [2] for several particular problems.

We demonstrate the strength and generality of the developed approach and formulate robust modifications of several popular algorithms, such as the Sample Matrix Inversion (SMI) [3], the diagonally loaded SMI (LSMI) [4], the Hung-Turner (HT) [5], and the Eigenvector Projection (EP) [6] methods. Although we consider only these algorithms, the developed approach allows to incorporate the robustness property into any other type of adaptive beamformer.

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2. DATA-DEPENDENT DERIVATIVE CONSTRAINTS

We formulate the problem for a linear array of n sensors taking into account that the approach considered is valid for planar or volume arrays, too. Let q ($q < n$) narrowband jammers and a single narrowband signal impinge on the array from unknown directions $\{\theta_1, \theta_2, \ldots, \theta_q\}$ and from a known direction θ_S, respectively. Let the jammers be uncorrelated with each other and with the signal, too. Unless otherwise specified, the jammer sources are assumed to be stationary (e.g. non-moving[1]) and the array output vectors are assumed to be statistically independent. Then the output vector at the i-th time instant (or snapshot) of the array can be expressed as:

$$\mathbf{y}(i) = \mathbf{A}\mathbf{s}(i) + \beta s_S(i)\mathbf{a}_S + \mathbf{n}(i), \quad \mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_q] \quad (1)$$

where the jammer direction vectors $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_q$, and desired signal vector $\mathbf{a}_S$ can be modelled as plane waves:

$$\mathbf{a}(\theta) = (\exp\{jx_1\xi\}, \exp\{jx_2\xi\}, \ldots, \exp\{jx_n\xi\})^T \quad (2)$$

Here $\mathbf{a}(\theta)$ is the $n \times 1$ direction vector corresponding to the angle θ, $\mathbf{a}_S = \mathbf{a}(\theta_S)$, $\xi = (2\pi/\lambda)\sin\theta$, λ is the wavelength, x_l is the coordinate of lth sensor, $\mathbf{s}(i) = (s_1(i), s_2(i), \ldots, s_q(i))^T$ is the $q \times 1$ vector of random jammer waveforms, and $s_S(i)$ is the signal waveform. The $n \times 1$ vector $\mathbf{n}(i)$ contains random sensor noise, while parameter β (0 or 1) indicates the presence of the signal. The $n \times n$ interference-plus-noise covariance matrix is

$$\mathbf{R} = \sigma^2\mathbf{I} + \mathbf{A}\mathbf{S}\mathbf{A}^H \quad (3)$$

where σ^2 is the noise variance, $\mathbf{S} = \mathrm{E}\{\mathbf{s}(i)\mathbf{s}^H(i)\}$ is the $q \times q$ covariance matrix of the jammer waveforms, $\mathbf{I}$ is the identity matrix, and $(\cdot)^H$ denotes Hermitian transpose.

The complex adaptive beamformer output with the adaptive weight vector $\mathbf{w}$ at the ith time instant is

$$z(i) = \mathbf{w}(i)^H\mathbf{y}(i)$$

In the stationary case and if $\mathbf{R}$ is known, the solution for the optimum weight vector of the adaptive array maximizing the signal-to-interference-plus-noise ratio (SINR) can be expressed in the well known form:

$$\mathbf{w}_{\mathrm{opt}} = \alpha\,\mathbf{R}^{-1}\mathbf{a}_S \quad (4)$$

where α is a main beam gain constant, which does not affect the output SINR and will be omitted below without loss of generality. Typically, jammers are much more powerful than the sensor noises and the signal. We use this assumption below.

[1] We use this assumption only for derivations and cancel it later.

Define an orthogonal projection on the column space of a $m_1 \times m_2$ full rank matrix $\mathbf{C}$ and an orthogonal projection onto its orthogonal complement as

$$\mathbf{P_C} = \mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{C}^H, \quad \mathbf{P}_\mathbf{C}^\perp = \mathbf{I} - \mathbf{P_C} \tag{5}$$

respectively. Here $m_1 > m_2$, and so $\mathbf{P_C}$ and $\mathbf{P}_\mathbf{C}^\perp$ are $m_1 \times m_1$ matrices.

Applying the matrix inversion lemma to the matrix $\mathbf{I} + \alpha\,\mathbf{CHC}^H$, where $\mathbf{H}$ is any $m_2 \times m_2$ non-singular matrix and $\alpha > 0$, gives the property $\mathbf{P}_\mathbf{C}^\perp = \lim_{\alpha\to\infty}\left(\mathbf{I} + \alpha\,\mathbf{CHC}^H\right)^{-1}$. Applying it to (3), we get $\lim_{\sigma^2\to 0}\sigma^2\,\mathbf{R}^{-1} = \mathbf{P}_\mathbf{A}^\perp = \mathbf{I} - \mathbf{P_A}$ where $\mathbf{P_A}$ is the orthogonal projection onto the jammer subspace. This implies that for strong jammers the adaptive weight vector (4) tends to be orthogonal to the jammer subspace, i.e.: $\lim_{\sigma^2\to 0}\mathbf{w}_{\rm opt} \perp \{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_q\}$. This means that the antenna pattern $\mathbf{w}^H\mathbf{a}(\theta)$ has nulls in the jammer directions. To broaden the null width we may require a higher order of the null, i.e. by requiring p-th order derivative constraints:

$$\partial^m(\mathbf{w}^H\mathbf{a}(\theta))/\partial\xi^m|_{\theta=\theta_k} = 0,$$
$$k = 1, 2, \ldots, q, \quad m = 1, 2, \ldots, p \tag{6}$$

Using (2), we can rewrite (6) as

$$\mathbf{w} \perp \{\mathbf{B}^m\mathbf{a}_i\}_{m=1,2,\ldots,p;\; i=1,2,\ldots,q} \tag{7a}$$

where

$$\mathbf{B} = \mathrm{diag}\{x_1, x_2, \ldots, x_n\} \tag{7b}$$

To fulfill these constraints, we assume $n > (p+1)q$.

In practice, neither $\mathbf{R}$, nor the jammer directions are known. The only available is the sequence of array snapshots $\mathbf{y}(i)$. However, for high interference-to-noise ratios (INR) and small signal a projection orthogonal to the array data is the same as a projection orthogonal to the jammer subspace. I.e., we obtain from (1) the asymptotic relation between a stationary ergodic snapshot sequence and the jammer subspace: $\lim_{\sigma^2\to 0,\, p_S\to 0}\mathbf{P_Y} = \mathbf{P_A}$, where p_S denotes the signal power and $\mathbf{Y}$ is the $n \times L$ data matrix consisting of $L \geq q$ statistically independent snapshot vectors in the columns [2] (as in (9b) below). With this property we can reformulate the constraints $\mathbf{w} \perp \{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_q\}$ and $\mathbf{w} \perp \{\mathbf{B}^m\mathbf{a}_i\}$, $m = 1, 2, \ldots, p$, $i = 1, 2, \ldots, q$ in an asymptotically ($\sigma^2 \to 0$, $p_S \to 0$) equivalent form:

$$\mathbf{w} \perp \mathbf{y}(i), \quad \mathbf{w} \perp \{\mathbf{B}^m\mathbf{y}(i)\}_{m=1,2,\ldots,p}, \quad \forall i \tag{8}$$

Equation (8) describes data-dependent derivative constraints that do not require any *a priori* knowledge of the jammer directions. For incorporation these constraints into any adaptive beamforming algorithm, one should use new "derivative" snapshots $\mathbf{B}^m\mathbf{y}(i)$, $m = 1, 2, \ldots, p$ in addition to the conventional data snapshots $\mathbf{y}(i)$ for the calculation of the weight vector.

3. ROBUST ALGORITHMS

3.1. SMI algorithm

The SMI algorithm computes the weight vector using the sample covariance matrix $\hat{\mathbf{R}}$:

$$\mathbf{w}(i) = \hat{\mathbf{R}}^{-1}(k)\mathbf{a}_S, \quad \hat{\mathbf{R}}(k) = \frac{1}{L}\mathbf{Y}(k)\mathbf{Y}(k)^H, \tag{9a}$$

[2] In the case $q < L$ and $\sigma^2 = 0$, $p_S = 0$ the matrix $\mathbf{Y}$ becomes rank deficient and the projection $\mathbf{P_Y}$ should not be calculated by (5), but correspondingly with q linear independent columns of $\mathbf{Y}$.

$$\mathbf{Y}(k) = [\mathbf{y}(k-L), \mathbf{y}(k-L+1), \ldots, \mathbf{y}(k-1)] \tag{9b}$$

The matrix $\hat{\mathbf{R}}(k)$ is invertible only if $L \geq n$ and $\sigma^2 > 0$. The relationship between the adaptation window and the beamforming snapshot, i.e. the choice of the parameter $k - i$, depends on the specific application of the algorithm and has of course a significant influence in non-stationary situations.

Let us consider asymptotic property of the weight vector (9a) for strong jammers. In the signal absent case and $L \geq q$ the sample covariance matrix can be written as

$$\hat{\mathbf{R}} = \sigma^2\mathbf{N} + \mathbf{AFA}^H$$

where $\mathbf{F}$ is the full rank $q \times q$ jammer sample covariance matrix and $\sigma^2\mathbf{N}$ is the $n \times n$ matrix containing the noise sample covariance matrix and jammer-noise sample covariance cross-terms. Applying the matrix inversion lemma, we have

$$\lim_{\sigma^2\to 0,\, p_S\to 0}\sigma^2\hat{\mathbf{R}}^{-1} = \mathbf{N}^{-1} - \mathbf{N}^{-1}\mathbf{A}(\mathbf{A}^H\mathbf{N}^{-1}\mathbf{A})^{-1}\mathbf{A}^H\mathbf{N}^{-1}$$

This is the oblique projection with respect to the Euclidean scalar product. For large number of snapshots all jammer-noise cross-terms vanish and $\lim_{L\to\infty}\mathbf{N} = \mathbf{I}$. This means that

$$\lim_{L\to\infty,\, \sigma^2\to 0,\, p_S\to 0}\sigma^2\hat{\mathbf{R}}^{-1} = \mathbf{P}_\mathbf{A}^\perp$$

Hence, the weight vector of the SMI method converges to the orthogonal projection of the desired signal vector onto the orthogonal complement of the jammer subspace. Therefore, the incorporation of new "derivative" snapshots into the sample covariance matrix means that the weight vector converges to the projection of the desired signal vector onto the orthogonal complement of the subspace spanned by vectors $\{\mathbf{B}^m\mathbf{a}_i\}$, $m = 0, 1, \ldots, p$, $i = 1, 2, \ldots, q$. Although any order of constraints is possible, we consider only first-order constraints (i.e. $p = 1$). The robust version of SMI algorithm can then be expressed as:

$$\mathbf{w}(i) = \tilde{\mathbf{R}}^{-1}(k)\mathbf{a}_S, \quad \tilde{\mathbf{R}}(k) = \hat{\mathbf{R}}(k) + \zeta\,\mathbf{B}\hat{\mathbf{R}}(k)\mathbf{B}$$
$$= \frac{1}{L}\left(\mathbf{Y}(k)\mathbf{Y}(k)^H + \zeta\,\mathbf{BY}(k)\mathbf{Y}(k)^H\mathbf{B}\right) \tag{10}$$

The real positive weight ζ controls the relative contribution of the "derivative" data.

3.2. LSMI algorithm

In the LSMI algorithm a small real positive number γ is added to the diagonal of the sample covariance matrix. This diagonal load warrants the sample covariance matrix invertibility in the case $L < n$, and reduces the variance of the adaptive weight vector. This allows to consider cases $q \leq L < n$, without too serious performance loss and makes the LSMI method well suited to non-stationary jammer situations, where the weights should follow the non-stationary jammers. The diagonally loaded sample covariance matrix is given by

$$\hat{\mathbf{R}}_{\rm DL}(k) = \gamma\mathbf{I} + \frac{1}{L}\mathbf{Y}(k)\mathbf{Y}(k)^H \tag{11}$$

It can be shown that for $L \geq q$,

$$\lim_{\gamma\to 0,\, \sigma^2\to 0,\, p_S\to 0}\gamma(\gamma\mathbf{I} + \hat{\mathbf{R}})^{-1} = \mathbf{P}_\mathbf{A}^\perp$$

Hence, the incorporation of new "derivative" snapshots into the matrix (11) corresponds asymptotically to constraining the derivatives of the adapted pattern. As in the modified SMI method, the weight vector will converge to the projection of the desired signal vector onto the orthogonal complement of the subspace spanned by vectors $\{\mathbf{B}^m \mathbf{a}_i\}$, $m = 0, 1, \ldots, p$, $i = 1, 2, \ldots, q$. However, the large number of snapshots condition is no longer necessary for the convergence to the orthogonal projection onto the orthogonal compliment to the jammer subspace. By this fact one can expect a better performance of the LSMI method for small/moderate number of snapshots than of the SMI method.

Following these considerations, we formulate the robust modification of the LSMI algorithm:

$$\mathbf{w}(i) = \tilde{\mathbf{R}}_{\mathrm{DL}}^{-1}(k)\mathbf{a}_S, \quad \tilde{\mathbf{R}}_{\mathrm{DL}}(k) = \hat{\mathbf{R}}_{\mathrm{DL}}(k) + \zeta\, \mathbf{B}\hat{\mathbf{R}}(k)\mathbf{B}$$

$$= \gamma \mathbf{I} + \frac{1}{L}\left(\mathbf{Y}(k)\mathbf{Y}(k)^H + \zeta\, \mathbf{B}\mathbf{Y}(k)\mathbf{Y}(k)^H \mathbf{B}\right) \qquad (12)$$

3.3. Hung-Turner algorithm

In this method, the property given by the 1st equation of (8) is used directly, so that

$$\mathbf{w}(i) = \mathbf{P}^{\perp}_{\mathbf{Y}(k)} \mathbf{a}_S \qquad (13)$$

The number of snapshots L is here also the dimension of the subspace on the complement of which is projected and should be chosen as $q \leq L < n$. For finite INR it is useful to choose $L > q$. If L is chosen too large, the jammers are well suppressed, but the gain of the antenna in the desired signal direction may be small. It was found in [7] that for large arrays $L \simeq 2q ... 3q$ gives a reasonable performance. The robust version of the HT algorithm using "derivative" snapshots up to the pth order can be written as:

$$\mathbf{w}(i) = \mathbf{P}^{\perp}_{\mathbf{Q}(k)} \mathbf{a}_S, \quad \mathbf{Q}(k) = [\mathbf{Y}(k), \mathbf{B}\mathbf{Y}(k), \ldots, \mathbf{B}^p \mathbf{Y}(k)] \qquad (14)$$

For a non-trivial projection $(p+1)L < n$. The robust algorithm (14) requires additional degrees of freedom as compared with (13). The choice of the optimum number of snapshots L is therefore linked to the order of the derivative constraints. For (14) no weight ζ is necessary.

3.4. Eigenvector projection algorithm

The eigendecomposition of $\hat{\mathbf{R}}(k)$ can be written as

$$\hat{\mathbf{R}}(k) = \sum_{i=1}^{n} \hat{\lambda}_i(k) \hat{\mathbf{u}}_i(k) \hat{\mathbf{u}}_i(k)^H$$

where $\hat{\lambda}_i(k)$, $i = 1, 2, \ldots, n$ $\quad (\hat{\lambda}_1(k) > \hat{\lambda}_2(k) > \cdots > \hat{\lambda}_n(k))$ are the ordered sample eigenvalues (distinct w. p. 1), $\hat{\mathbf{u}}_i(k)$ is the sample eigenvector that corresponds to $\hat{\lambda}_i(k)$. The weight vector in EP algorithm is calculated as the orthogonal projection of the desired signal vector onto the orthogonal complement to the subspace spanned by M dominant eigenvectors:

$$\mathbf{w}(i) = \mathbf{P}^{\perp}_{\mathbf{U}(k)} \mathbf{a}_S, \quad \mathbf{U}(k) = [\hat{\mathbf{u}}_1(k), \hat{\mathbf{u}}_2(k), \ldots, \hat{\mathbf{u}}_M(k)] \qquad (15)$$

In moving jammer case it is useful to choose $M > q$. From the analogy to the HT algorithm, the robust modification of the EP algorithm (15) can be formulated as:

$$\mathbf{w}(i) = \mathbf{P}^{\perp}_{\mathbf{V}(k)} \mathbf{a}_S, \quad \mathbf{V}(k) = [\mathbf{U}(k), \mathbf{B}\mathbf{U}(k), \ldots, \mathbf{B}^p \mathbf{U}(k)] \qquad (16)$$

As the robust HT algorithm, this algorithm requires the additional degrees of freedom and the choice of the optimum M is linked to the order p of the derivative constraints.

3.5. Optimization of derivative constraint weight

Let us discuss the choice of the parameter ζ in robust SMI and LSMI methods. In terms of the jammer subspace, the derivative constraints have the effect of additional jammers. If the contribution of the "derivative" data vectors is too strong as compared with that of the original data, the depth of the nulls is not sufficient and, as a result, the jamming power is not sufficiently suppressed. Conversely, when the contribution of the original data is much stronger than that of "derivative" data, the desired null width may be not sufficient. The choice of the parameter ζ in (10), (12) is therefore very important for the optimization of the adaptive array performance. Let us find a value of ζ from the compromise between null depth and width of the adapted pattern. With respect to moving jammer scenarios the robustness should improve with increasing ζ. However, one cannot expect a better result than in a stationary jammer situation. Therefore, the maximal value of the parameter ζ should be limited from above by a maximally admissible loss in SINR in the stationary case, i.e. ζ should be chosen as a highest value satisfying

$$\mathrm{SINR}_{\mathrm{opt}} / \mathrm{SINR}_{\mathrm{rob}} \leq \Pi \qquad (17)$$

The loss $\Pi \geq 1$ is the price we accept to pay for the improved robustness. $\mathrm{SINR}_{\mathrm{opt}}$ is the SINR with the weight (4), while $\mathrm{SINR}_{\mathrm{rob}}$ corresponds to $\mathbf{w}_{\mathrm{rob}} = (\mathbf{R} + \zeta\, \mathbf{B}\mathbf{R}\mathbf{B})^{-1}\mathbf{a}_S$. The SINR for an arbitrary weight vector $\mathbf{w}$ is defined as $\mathrm{SINR} = p_S |\mathbf{w}^H \mathbf{a}_S|^2 / \mathbf{w}^H \mathbf{R}^{-1} \mathbf{w}$. From the matrix inversion lemma

$$\mathbf{w}_{\mathrm{rob}} = (\mathbf{R}^{-1} - \mathbf{R}^{-1}\mathbf{B}(\zeta^{-1}\mathbf{R}^{-1} + \mathbf{B}\mathbf{R}^{-1}\mathbf{B})^{-1}\mathbf{B}\mathbf{R}^{-1})\mathbf{a}_S$$

Hence, the optimum choice of ζ depends on unknown jammer parameters. However, with the following assumptions we can estimate a value of ζ independent of these parameters:

1. $\mathbf{R}^{-1} \simeq \mathbf{P}^{\perp}_{\mathbf{A}}$. This means that we have strong jammers and assume (without loss of generality) $\sigma^2 = 1$.

2. $\mathbf{P}^{\perp}_{\mathbf{A}} \mathbf{a}_S \simeq \mathbf{a}_S$. In other words, let the jammers impinge from sidelobe directions, i.e. $|\mathbf{a}_S^H \mathbf{a}_i|^2 \ll \mathbf{a}_S^H \mathbf{a}_S\, \mathbf{a}_i^H \mathbf{a}_i = n^2$, for $i = 1, 2, \ldots, q$. This is given, if $n \gg q$ and then $||\mathbf{P}_{\mathbf{A}} \mathbf{a}_S|| \ll ||\mathbf{a}_S||$, hence $\mathbf{P}^{\perp}_{\mathbf{A}} \mathbf{a}_S \simeq \mathbf{a}_S$.

3. $\mathbf{P}^{\perp}_{\mathbf{A}} \mathbf{B} \simeq \mathbf{B}$. This condition can be approximately fulfilled for large, centered arrays ($n \gg q$, $\sum x_l = 0$). Then the vectors $\mathbf{a}_i$ and $\mathbf{B}\mathbf{a}_i$ are exactly orthogonal for all i. Further, if the jammers are well separated, such that $|\mathbf{a}_i^H \mathbf{B} \mathbf{a}_k|^2 \ll \mathbf{a}_i^H \mathbf{a}_i\, \mathbf{a}_k^H \mathbf{B}^2 \mathbf{a}_k = n \sum x_l^2$, for $i, k = 1, \ldots, q$, then this means that $\mathbf{A}^H \mathbf{B} \mathbf{A} \simeq \mathbf{O}$, from which $\mathbf{P}^{\perp}_{\mathbf{A}} \mathbf{B} \simeq \mathbf{B}$ follows.

Under these assumptions we can rewrite (17) as:

$$\mathbf{a}_S^H \mathbf{a}_S\, [\mathbf{a}_S^H \mathbf{a}_S - 2\mathbf{a}_S^H \mathbf{B}(\frac{1}{\zeta}\mathbf{I} + \mathbf{B}^2)^{-1}\mathbf{B}\mathbf{a}_S$$

$$+ \mathbf{a}_S^H \mathbf{B}(\frac{1}{\zeta}\mathbf{I} + \mathbf{B}^2)^{-1}\mathbf{B}^2(\frac{1}{\zeta}\mathbf{I} + \mathbf{B}^2)^{-1}\mathbf{B}\mathbf{a}_S]$$

$$\times [\mathbf{a}_S^H \mathbf{a}_S - \mathbf{a}_S^H \mathbf{B}(\frac{1}{\zeta}\mathbf{I} + \mathbf{B}^2)^{-1}\mathbf{B}\mathbf{a}_S]^{-2} \leq \Pi \qquad (18)$$

For a given array, a suitable value of ζ can be found from (18) which depends on the array geometry only. Assuming centered array and normalizing $\mathbf{B}$, such that $||\mathbf{B}\mathbf{a}|| = ||\mathbf{a}||$ for any direction vector $\mathbf{a}$, we have that instead of x_l we should use x_l/ρ with $\rho = \sqrt{\sum_{l=1}^{n} x_l^2 / n}$. In this case, the left-hand part of (18) can be further simplified, and one can find that it gives a monotone curve which is practically independent of n [8], [9].

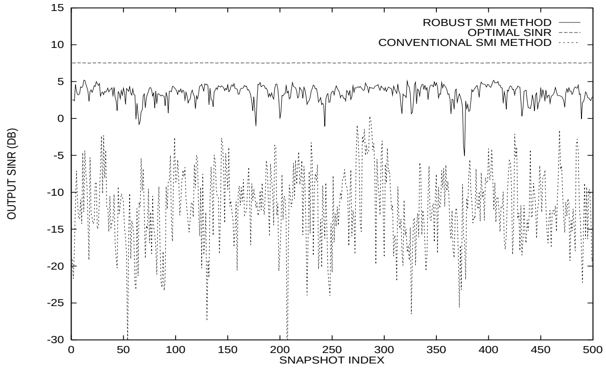

Figure 1: Performance of SMI methods with $L = 32$.

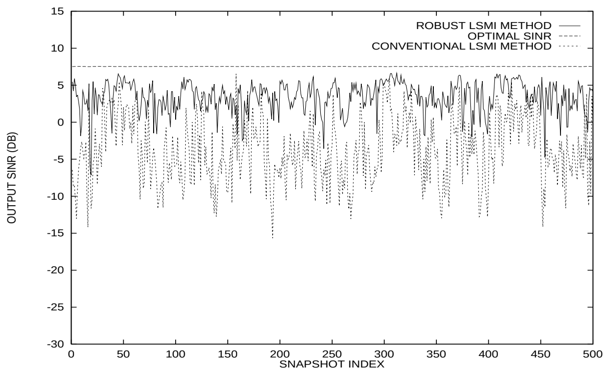

Figure 2: Performance of LSMI methods with $L = 10$.

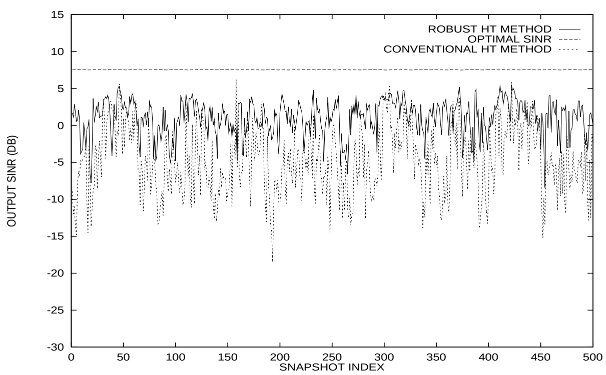

Figure 3: Performance of HT methods with $L = 10$.

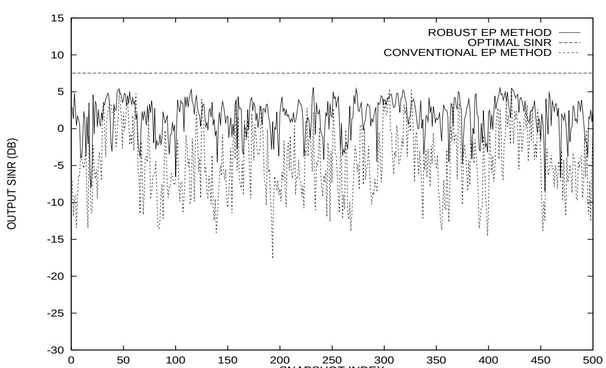

Figure 4: Performance of EP methods with $L = 10$, $M = 9$.

4. SIMULATION RESULTS

In simulations we assumed three uncorrelated moving narrowband jammers and a ULA of 32 sensors at $\lambda/2$ spacing. The simulated trajectories of angular jammer motion are:

$$\theta_1(i) = 20^\circ + 5^\circ \sin(i/15), \quad \theta_2(i) = -40^\circ - 10^\circ \cos(i/15),$$

$$\theta_3(i) = -25^\circ + 10^\circ \cos(i/20)$$

The INR was set to 30 dB for each jammer, and the SNR was set to -7.5 dB, in each sensor, respectively. We assumed an adaptation period without the desired signal present, i.e. a learning period with $\beta = 0$ in (1), followed by the beamforming snapshot with the desired signal present (i.e. $k - i = 0$ in (9a), (10), (12)–(16)). This scheme is representative for active systems[3]. In simulations, we assumed $\zeta = 1.5$ which corresponds to $\simeq 1$ dB performance loss in the stationary case. The order of the derivative constraints was always $p = 1$. Fig. 1 show the output SINR of the robust and conventional SMI algorithms with $L = 32$ compared with the optimal SINR. The similar curves showing the performance of LSMI, HT, and EP algorithms with $L = 10$ are shown in Figs. 2-4, respectively. For EP algorithms, $M = 9$ is taken.

Simulation results demonstrate that the performances of the conventional algorithms completely fail in moving jammer environment, while the proposed robust algorithms perform well in the presence of moving jammers. The proposed approach can be also efficiently exploited in the situation of rapid fluctuations or abrupt changes of jammer DOA's.

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[3] The examples representative for passive systems can be found in [8], [9], where the signal was assumed to be always present in data snapshots.